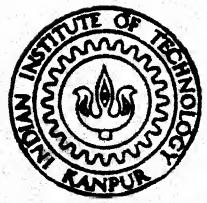
HOPF BIFURCATION AND LIMIT CYCLES IN FISSION REACTOR DYNAMICS

by

MANMOHAN PANDEY



DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JULY, 1988

HOPF BIFURCATION AND LIMIT CYCLES IN FISSION REACTOR DYNAMICS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

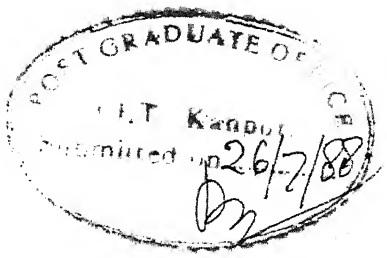
by

MANMOHAN PANDEY

to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JULY, 1988



CERTIFICATE

This is to certify that this work entitled, "HOPF BIFURCATION AND LIMIT CYCLES IN FISSION REACTOR DYNAMICS" has been carried out by Manmohan Pandey under my supervision and has not been submitted elsewhere for a degree.

M.S.Kalra

(M.S. Kalra)
Assistant Professor
Department of Mechanical Engineering
Indian Institute of Technology
KANPUR - 208016 (U.P.)

ABSTRACT

Dynamical systems for the nuclear fission reactors based on point reactor kinetics with reactivity feedback effects have been studied using modern concepts from nonlinear dynamics.

The governing equations were suitably transformed so as to minimize the number of parameters. The possibility of the existence of limit cycles for the various models was then investigated by analytical methods as well as numerical experiments. Limit cycles were ruled out in one-temperature models in the neighborhood of the operating point. In the two-temperature model without delayed neutrons, a limit cycle was predicted using the method of Hopf bifurcation. This was confirmed by numerical experiments, thus leading to the conclusion that the predicted cycle was stable. The necessary condition for Hopf bifurcation was also established in the two-temperature model with delayed neutrons.

Some additional investigations on the possibility of chaos were also done by computing the largest Lyapunov exponent. The result was in the negative for the particular parameter values suitably chosen after the Hopf bifurcation in the two-temperature model without delayed neutrons.

CONTENTS

CHAPTER I

INTRODUCTION

To begin with, in this chapter we introduce the subject, followed by a review of literature and an outline of the present work.

1.1. Prologue

Having created a giant in the form of nuclear fission reactor, man has to make sure it serves him as an obedient and efficient servant, without becoming a threat to its own master. Safe design and control of nuclear reactors necessitates an understanding of the transient behavior of reactor state variables, particularly the neutron population and fuel temperature, with respect to various parameters. This is the subject of study of nuclear reactor dynamics.

By mathematical modeling of a nuclear fission reactor, we get some equations which form a nonlinear dynamical system. Linear analysis of such a dynamical system only tells us about the initial effect of small disturbances about the operating point. In order to have a better understanding of the dynamics, we have to go for nonlinear analysis and bifurcation theory using state space concepts. Nonlinear analysis shows us more interesting orbits in state space than the linear analysis, which shows only stable and unstable fixed points, that too, only locally. Nonlinear analysis of a system may show periodic orbits (limit cycles), quasiperiodic orbits (tori), and aperiodic or chaotic orbits (strange attractors). By studying bifurcations, we may find out whether

such orbits occur in a particular dynamical system, and if so, at what values of the parameters.

If, for certain values of parameters, a disturbance from the operating point is going to take it to a limit cycle, ending up with ceaseless oscillations in neutron population, fuel temperature, etc., then such a phenomenon is certainly worth studying. Hence the study of limit cycles in nuclear reactor dynamics. A limit cycle is an isolated periodic solution of an autonomous system (i.e. a system without explicit dependence on time) represented in the state space by an isolated closed path. The neighboring paths are not closed, but spiral into or away from the limit cycle. The study of Hopf bifurcation provides us with a clue to the existence of limit cycles. In this type of bifurcation, a pair of complex conjugate eigenvalues of the Jacobian matrix crosses the imaginary axis transversely, i.e. with nonzero velocity, and makes it possible for a limit cycle to exist.

A limit cycle may further bifurcate to a 2-torus, i.e., a quasi-periodic or almost periodic orbit with two incommensurate frequencies, by undergoing a secondary Hopf bifurcation. Still further bifurcations may lead to a 3-torus and subsequently to chaos. Moreover, a system may go to chaos via other routes. The study of chaotic dynamics is very complicated. In this work we confine our scope mainly to the study of Hopf bifurcation and limit cycles in fission reactor dynamics, which is of great practical importance.

1.2 Time Dependent Modeling of Nuclear Fission Reactors

In nuclear reactor dynamics, a mathematical model of the nuclear reactor is obtained [1-4] in the form of partial differential equations describing the dependence of state variables, such as neutron population, reactor temperature etc., on space and time, and parameters. There are a number of methods for deriving reactor kinetics equations : they range from the rather complicated to the very simple [1].

1.2.1. Space Dependent Dynamic Model :

The equation for time dependent diffusion of neutrons in a reactor is [3]

$$\frac{\partial N}{\partial t} = Dv\ \nabla^2 N - \Sigma_a\ vN + S$$

where,

$N(\underline{r},t)dV$ = number of neutrons in a volume element dV at a point $\underline{r}$ at time t ;

$Dv\ \nabla^2 N dV$ = number of neutrons diffusing into dV per unit time at time t ;

$\Sigma_a vN dV$ = number of neutrons absorbed in dV per unit time at time t ;

$S(\underline{r},t)dV$ = number of neutrons produced in dV per unit time at time t. ;

D = diffusion constant ;

Σ_a = macroscopic neutron absorption cross-section. ;

v = neutron speed.

Here, it has been assumed that the neutron current density is given by Fick's

law, $\underline{j} = -Dv\,\nabla N$, and that all the coefficients are independent of position, and that their numerical values represent suitable averages over the neutron velocity distribution. Let,

k_∞ = infinite medium reproduction factor,

β_i = delayed neutron fraction for the i-th precursor,

$\beta = \Sigma_i \beta_i$,

λ_i = decay constant of the i-th precursor,

$C_i(\underline{r},t)$ = volumetric density of the i-th type of precursor.

Including sources of neutrons extraneous to the fission processes as a term $S_o(\underline{r},t)$, we get diffusion equation and the delayed neutron equation as [3].

$$\frac{\partial N}{\partial t} = Dv\,\nabla^2 N - \Sigma_a vN + (1-\beta)\,k_\infty \Sigma_a vN + \Sigma_i \lambda_i C_i + S_o \quad (1.2)$$

$$\frac{\partial C_i}{\partial t} = \beta_i k_\infty \Sigma_a vN - \lambda_i C_i \quad (1.3)$$

Thus we get a set of coupled partial differential equations, i.e. an infinite-dimensional dynamical system. To reduce it to a finite-dimensional system, i.e., a set of ordinary differential equations, we use the lumped parameter or point reactor model.

1.2.2. The Point Model of Reactor Kinetics :

In this model, it is assumed that all but the fundamental spatial harmonics of the flux, following a flux disturbance, have been attenuated in a very short time (a few microseconds) compared to times of reactor kinetics

interest [1]. That is, the neutron density in the reactor is now assumed to rise or fall as a whole following a reactivity change. Hence all spatial dependencies factor away, leaving time as the only remaining independent variable.

Assuming that N and C_i are separable in space and time, we substitute [3]

$$N(\underline{r},t) = f(\underline{r})\, n(t)\ ;\ C_i(\underline{r},t) = g_i(\underline{r})\, c_i(t) \tag{1.4}$$

into Eqs. (1.2) - (1.3). Then the removal of space dependence from the resulting equations requires that f/g_i, $\nabla^2 f/f$ and S_o/f be independent of position. Assuming

$$g_i = f;\ \nabla^2 f + B^2 f = 0$$

(where B^2 is the fundamental mode buckling) and letting

$$q(t) = S_o(\underline{r},t)/f(\underline{r}),$$

we get the point reactor model as

$$\frac{dn}{dt} = \frac{\rho - \beta}{l}\, n + \Sigma_i\, \lambda_i\, c_i + q \tag{1.5}$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l}\, n - \lambda_i\, c_i \tag{1.6}$$

where,

ρ = reactivity,

l = neutron generation time, and

q = effective source strength.

At high power, we may ignore the term q in Eq. (1.5) [3]. Further, we

use one group representation, i.e., we assume that all the delayed neutrons have a single mean lifetime. Then we get the equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{l} n + \lambda c \tag{1.7}$$

$$\frac{dc}{dt} = \frac{\beta}{l} n - \lambda c \tag{1.8}$$

This is the simplified point kinetic model which is a system of two coupled ordinary differential equations. The inclusion of feedback effects will make the first equation nonlinear. This will be discussed in detail in Chapter 2.

1.2.3. Discrete Models :

A further simplification in the finite dimensional continuous dynamical system of the point reactor model may be attained by discretizing the equations, or by starting with discrete population dynamics models. Such discrete models are the ones which are used in biological population studies, and frequently exhibit complicated or chaotic behavior. It is an open question whether or not discrete models are more appropriate than the continuous models in the study of neutron population in a nuclear reactor. We will have more to say about this in a later chapter.

1.3 Literature Review

Several papers have appeared [5-13] on the stability analysis and oscillatory behavior in nuclear reactor dynamics. Reator kinetics books [1-4]

have also discussed these aspects of nuclear reactor dynamics.

1.3.1. Books :

According to Ash [1], it is of paramount importance for stability determination to ascertain the existence of limit cycles, however, there seems to be no general way to determine whether or not a limit cycle exists. Earlier boosk by Lewins [2] and Hetrick [3] have shown the existence of periodic orbits in the one-temperature feedback model by neglecting the effect of delayed neutrons and assuming a constant power removal during the transient. These periodic orbits form a nonlinear center rather than limit cycles. Akcasu et al [4] have shown that in some reactor models, the reactor power does not return to its equilibrium value following an initial perturbation, nor does it diverge from it at a finite or infinite time. Instead, it oscillates between two limits about the equilibrium power level as time unfolds. They have called such solutions of the kinetic equations as bounded solutions or limit cycles.

1.3.2. Research Papers :

Akcasu and Shotkin [5] have predicted the existence of limit cycles for reactors with arbitrary linear feedback and no delayed neutrons, but they have admitted that proving the existence of limit cycles convincingly is a delicate problem. They have argued that when a reactor is linearly unstable but bounded, closed periodic curves (limit cycles) must exist. Devooght and Smets [6] have considered the determination of regions of stability. Schmidt

of stable and unstable limit cycles in two-temperature feedback model without and with delayed neutrons, respectively. Shotkin et al [8] have shown that the presence of delayed neutrons may result in unstable limit cycles. Extending the approach of Akcasu and Shotkin [5], they have predicted the existence of an unstable limit cycle by showing that the system is locally stable but not stable in the large. Vreeke and Sandquist [9] have demonstrated the existence of stable limit cycles in two-temperature feedback model without delayed neutrons by phase space computations. Keener and Cohen [10] have analysed the same model as in Ref. [9] using a perturbation method and shown the existence of stable limit cycles. Poddar et al [11] have analysed limit cycles using perturbation techniques ignoring the presence of delayed neutrons.

In most of the above works, the presence of delayed neutrons was ignored. Ashok Kumar [12] analysed the two-temperature feedback model with delayed neutrons. Using a perturbation approach for linearization, he established the conditions for oscillatory and non-oscillatory transient behavior as well as sustained oscillations, and verified the results of this linear analysis by numerical computations. Enginol [13] has obtained a criterion for asymptotic stability of a reactor in terms of the equilibrium power level and delayed neutron parameters.

1.4. Outline of the Present Work

In the literature cited, modern concepts of nonlinear dynamics such as Hopf bifurcation have not been used to predict the existence of limit

cycles and analyse their stability. The present work attempts to fill this gap.

In Chapter 2, feedback effects are included in the equations of point-reactor kinetics. The dynamic equations thus obtained are transformed to a dimensionless form, and dynamical systems for various reactor models are constructed.

The dynamical systems constructed in Chapter 2, are explored for the possibility of Hopf bifurcation and the existence of limit cycles in Chapter 3. The predictions based on the analysis are verified by numerical experiments.

Some additional investigations on these systems, to see the possibility of chaos, are reported in Chapter 4.

Finally, Chapter 5 concludes the thesis with a summary of the present work and some suggestions for further work.

* * *

CHAPTER 2

CONSTRUCTION OF DYNAMICAL SYSTEMS FOR NUCLEAR FISSION REACTORS

In this chapter, the point reactor model with energy balances and reactivity feedback is formulated. These equations are put in a convenient form by a suitable translation and normalization. Then dynamical systems for various models are constrcuted by making some approximations.

2.1. The Lumped Parameter Dynamic Model

In Section 1.2.2, we obtained Eqs. (1.7) and (1.8) for the point reactor model with one group of delayed neutrons. Writing reactor power, p instead of neutron density n, and using power units for c also, we can rewrite the equations as

$$\frac{dp}{dt} = \frac{\rho - \beta}{l} p + \lambda c \tag{2.1}$$

$$\frac{dc}{dt} = \frac{\beta}{l} p - \lambda c \tag{2.2}$$

To these equations, we shall now append the equations obtained from energy balances and include their feedback effect in the first equation.

2.1.1. Energy Balances :

The lumped parameter thermal model is based on two energy balances: one for the energy contained in all the fuel elements of the reactor core,

and another for the energy stored in the coolant within the core volume.

The energy balance for the fuel elements is given by

$$C_f \frac{dT_f}{dt} = p - h_f (T_f - T_c)$$

where,

C_f = heat capacity of the fuel elements in the reactor core (JK^{-1}),

h_f = fuel to coolant heat transfer coefficient (WK^{-1}),

T_f, T_c = average temperatures of the fuel and the coolant, respectively.

Dividing all the terms by C_f, we can rewrite the equation as

$$dT_f/dt = Kp - \gamma (T_f - T_c) \qquad (2.3)$$

where, $K = 1/C_f$ and $\gamma = h_f/C_f$.

The energy balance for the coolant within the core is given by

$$C_c \frac{dT_c}{dt} = h_f(T_f - T_c) - h_c(T_c - T_i)$$

where,

C_c = heat capacity of the coolant within the reactor core (JK^{-1}),

T_i = coolant inlet temperature, and

h_c = twice the product of the mass flow rate and specific heat of the coolant (WK^{-1}).

Dividing all the terms by C_c, we can rewrite the equation as

$$dT_c/dt = \gamma r(T_f - T_c) - s(T_c - T_i) \quad (2.4)$$

where, $r = C_f/C_c$, $s = h_c/C_c$, and γ is as defined in Eq. (2.3).

2.1.2. Reactivity Feedback Effects :

The reactivity $\rho(t)$ that drives a transient is the net effect of contributions arising from the external disturbances to which the transient is due and the termal hydraulic feedback effects. Therefore, it can be written as

$$\rho(t) = \rho_i(t) + \rho_{fb}(t)$$

where $\rho_i(t)$ and $\rho_{fb}(t)$ are the reactivities from the external disturbances and thermal hydraulic feedback effects, respectively. Assuming linear feedback, we write

$$\rho_{fb}(t) = \alpha_f(T_f - T_f^R) + \alpha_c(T_c - T_c^R)$$

where,

α_f, α_c = fuel and coolant temperature coefficients of reactivity, respectively, and

T_f^R, T_c^R = some reference values of T_f, T_c respectively.

These reference temperatures may be chosen arbitrarily. If we choose them as the steady state operating temperatures T_f^o and T_c^o, then ρ_i turns out to be zero. Therefore, for the linear feedback model, we have

$$\rho = \alpha_f(T_f - T_f^o) + \alpha_c(T_c - T_c^o) \quad (2.5)$$

For higher order feedback, we can write

$$\rho = \alpha_f(T_f - T_f^o) + \alpha_c(T_c - T_c^o)$$

$$+ F(T_f - T_f^o, T_c - T_c^o) \qquad (2.6)$$

where F (x, y) is a nonlinear function with no constant or linear terms, i.e.

$$F(o,o) = o\ ;\ F_x(o,o) = F_y(o,o) = o$$

2.2. Derivation of Dimensionless Equations

Eqs. (2.1) - (2.4) appended by Eq. (2.5) represent the lumped parameter dynamic model with linear reactivity feedback. It has two fixed points : the shut down point (0,0, T_i, T_i) and the operating point (p_o, c_o, T_f^o , T_c^o), where p_o and c_o are the steady state operating values of p and c, respectively, in the final system (after the disturbances). Now we obtain dimensionless form of these equations with appropriate normalization and translation.

2.2.1. Normalization of State Variables :

In choosing the normalization and translation of the state variables, the considerations were :

(1) The resulting transformed state variables should be dimensionless,

(2) The normalization should not be done with respect to parameters passing through zero,

(3) The number of parameters in the final equations should be minimum,

(4) The operating point should be brought to the origin.

(5) The shutdown point should be independent of the parameters, if possible.

(6) The transformed variables and the parameters should allow some direct physical interpretation, as far as possible.

Several types of normalization and translation were tried, and, on the basis of the above criteria, the following set was chosen :

$$X_1 = \frac{(P - P_o)}{P_o} \quad , \quad X_2 = \frac{(C - C_o)}{C_o} \tag{2.7}$$

$$X_3 = \frac{T_f - T_f^o}{T_f^o - T_i} \quad , \quad X_4 = \frac{T_c - T_c^o}{T_f^o - T_i} \tag{2.8}$$

2.2.2. Time Normalization :

In deciding time normalization, the criteria were :

(1) A dimensionless time should be obtained.

(2) The number of parameters in the equations should be reduced to a minimum.

(3) The orders of magnitude of the final parameters should be reasonable.

(4) The time scale should not become too inflated, so as to require too large integration steps.

With these considerations, several time normalizations were tried, and the following one, with respect to the decay constant, was chosen :

$$\tau = \lambda t \tag{2.9}$$

2.2.3. The Dimensionless Equations :

Making the aforementioned transformations in Eqs. (2.1) to (2.5), we get the dimensionless form of the equations as :

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + qX_4 + aX_1X_3 + qX_1X_4 \tag{2.10}$$

$$\dot{X}_2 = X_1 - X_2 \tag{2.11}$$

$$\dot{X}_3 = k(1-p)\,X_1 - kX_3 + kpX_4 \tag{2.12}$$

$$\dot{X}_4 = \frac{kr}{p}\,X_3 - \frac{kr}{p}X_4 \tag{2.13}$$

where overhead dots denote derivatives with respect to t. This system has six parameters :

$$b = \beta / \lambda l \;,\; a = \alpha_f(T_f^o - T_i) / \lambda l,\; q = \alpha_c(T_c^o - T_i)/\lambda l \;,$$

$$k = \gamma/\lambda = h_f/C_f\lambda = KP_o / \lambda\,(T_f^o - T_c^o),$$

$$p = (T_c^o - T_i) / (T_f^o - T_i)\;,\quad r = C_f/ C_c = 1/KC_c \;,$$

where the various quantities appearing on the right hand sides have already been defined, and all the new parameters, as well as the dependent and independent variables are dimensionless.

The system of Eqs. (2.10) - (2.13) has two fixed points : the operating point (0,0,0,0) and the shutdown point (-1, -1, -1, -1). This suggests that the values of state variables below - 1 would be unrealistic. Using the equations with appropriate approximations, we can construct dynamical systems for various types of point reactor models.

2.3 Two-Temperature Feedback Models

The thermal model used in the previous sections was based on two-temperature feedback via two energy balances. Delayed neutrons were included and the feedback was assumed to be linear .

2.3.1. Model with Delayed Neutrons :

This is the model already given by Eqs. (2.10) to (2.13). Defining for convenience, new parameters,

$$h = kp,\ f = kr/p,$$

we can rewrite the equations as :

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + qX_4 + aX_1X_3 + qX_1X_4 \qquad (2.14)$$

$$\dot{X}_2 = X_1 - X_2 \qquad (2.15)$$

$$\dot{X}_3 = (k-h)\,X_1 - kX_3 + hX_4 \qquad (2.16)$$

$$\dot{X}_4 = fX_3 - fX_4 \qquad (2.17)$$

This is a four-dimensional continuous dynamical system with six parameters and two fixed points (0,0,0,0) and (-1, -1, -1, -1).

2.3.2. Model without Delayed Neutrons :

To obtain this model, we simply put b = 0 in Eq. (2.14) and forget about Eq. (2.15). Thus we get the model :

$$\dot{X}_1 = aX_3 + qX_4 + aX_1X_3 + qX_1X_4 \qquad (2.18)$$

$$\dot{X}_2 = (k-h)\, X_1 - kX_2 + hX_3 \tag{2.19}$$

$$\dot{X}_3 = fX_2 - fX_3 \tag{2.20}$$

where we have written X_2 and X_3 respectively, in place of X_3 and X_4 of the previous system. It is understood that in the calculation of the parameter a, the value of l will be suitably changed to an effective lifetime so as to compensate, as far as possible, for ignoring delayed neutrons.

This is a three-dimensional continuous dynamical system with five parameters and two fixed points (0,0,0) and (-1, -1, -1).

2.3.3. Higher Order Feedback Models :

So far, we have assumed the reactivity feedback to be linear. To include the effect of higher order feedback, we transform Eqs. (2.1) to (2.4) in the same manner as before, using Eq. (2.6) instead of Eq. (2.5). Thus, we can replace Eq. (2.14) by

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + qX_4 + aX_1X_3 + qX_1X_4 + (1+X_1)\,\psi(X_3,X_4) \tag{2.21}$$

where ψ is related to the nonlinear term F in Eq. (2.6). It can be easily seen that

$$\psi(0,0) = 0, \quad \frac{\partial \psi}{\partial X_3}\Big|_{(0,0)} = 0 \text{ and } \frac{\partial \psi}{\partial X_4}\Big|_{(0,0)} = 0$$

Similarly, Eq. (2.18) (in case of absence of delayed neutrons) can be replaced by

$$\dot{X}_1 = aX_3 + qX_4 + aX_1X_3 + qX_1X_4 + (1+X_1)\,\psi(X_3,X_4) \tag{2.22}$$

The models with nonlinear feedback have the same sets of parameters and fixed points as those with linear feedback, with the addition of a nonlinear function.

2.4. One-Temperature Feedback Models

The one-temperature feedback model is based on the assumption that $\alpha_c = 0$ and that the heat capacity of the coolant within the core, C_c, is negligibly small, so that, it must transfer away all the heat it receives from the fuel elements. Therefore, in Eq. (2.13), we take the limit as $r \to \infty$ and obtain the relation $X_3 = X_4$. Substituting this in Eq. (2.12), we get

$$\dot{X}_3 = k(1-p)\, X_1 - k(1-p)\, X_3$$

or

$$\dot{X}_3 = cX_1 - cX_3 \tag{2.23}$$

where,

$$c = k(1-p) = \frac{\gamma (T_f^o - T_c^o)}{\lambda (T_f^o - T_i)} = \frac{Kp_o}{\lambda (T_f^o - T_i)} = \frac{p_o}{h_f (T_f^o - T_i)}$$

Thus, we have eliminated one equation by removing the coolant temperature from the set of state variables. In reactivity feedback also, we consider the effect of only the reactor fuel temperature, and take the following equation intead of Eq. (2.5) :

$$\rho = \alpha_f (T_f - T_f^o) \tag{2.24}$$

so that Eq. (2.10) is replaced by

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + aX_1X_3 \tag{2.25}$$

2.4.1. Model with Delayed Neutrons :

The equations for this model are :

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + aX_1X_3 \tag{2.26}$$

$$\dot{X}_2 = X_1 - X_2 \tag{2.27}$$

$$\dot{X}_3 = cX_1 - cX_3 \tag{2.28}$$

This is a three-dimensional continuous dynamical system with three parameters and two fixed points (0,0,0) and (-1, -1, -1).

2.4.2. Model without Delayed Neutrons :

As in Section 2.3.2, here also we put b = 0 in Eq. (2.26) and delete Eq. (2.27), thus obtaining :

$$\dot{X}_1 = aX_2 + aX_1X_2 \tag{2.29}$$

$$\dot{X}_2 = cX_1 - cX_2 \tag{2.30}$$

where we have written X_2 in place of X_3 of the previous system. It is understood that in the calculation of the parameter a, the value of l will be changed so as to partially compensate for ignoring delayed neutrons.

This is a two-dimensional continuous dynamical system with two parameters and two fixed points (0,0) and (-1, -1).

2.4.3. Model without Delayed Neutrons and with Constant Power Removal (Fast Transients) :

In this model, it is assumed that heat from the reactor fuel elements

is removed at a constant rate, equal to the equilibrium power. This results in dropping out of the X_2 term in Eq. (2.30), and we get the system :

$$\dot{X}_1 = aX_2 + aX_1X_2 \tag{2.29}$$

$$\dot{X}_2 = cX_1 \tag{2.31}$$

This system is two-dimensional and has only one fixed point i.e. (0,0), and two parameters.

2.4.4. Higher-Order Feedback Models :

For one-temperature higher-order feedback, we use the following equation for reactivity feedback (instead of the linear Eq. (2.24)) :

$$\rho = \alpha_f(T_f - T_f^o) + f(T_f - T_f^o) \tag{2.32}$$

where f(x) is a nonlinear function with

$$f(0) = f'(0) = 0.$$

Now, instead of Eq. (2.25), we get

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + aX_1X_3 + (1 + X_1)\,\phi(X_3) \tag{2.33}$$

where ϕ is related to the function f in Eq. (2.32) and has the properties :

$$\phi(0) = \phi'(0) = 0.$$

The models with higher-order feedback have the same sets of parameters and fixed points as those with linear feedback.

2.5. Direct Feedback Models

This model is a further simplification of the one-temperature feedback model. Here we remove the reactor temperature from the set of state variables so that a change in reactor power directly causes a feedback effect. To do this, we take the limit as $\dot{X}_3 \to 0$ (very slow transients) in Eq. (2.23) and obtain $X_1 = X_3$. Substituting this in Eq. (2.25), we get

$$\dot{X}_1 = (a-b)\, X_1 + bX_2 + aX_1^2 \tag{2.34}$$

2.5.1. Model with Delayed Neutrons :

This model is given by

$$\dot{X}_1 = (a-b)\, X_1 + bX_2 + aX_1^2 \tag{2.35}$$

$$\dot{X}_2 = X_1 - X_2 \tag{2.36}$$

This system is two-dimensional with two parameters and two fixed points (0,0) and (-1,-1).

2.5.2. Model without Delayed Neutrons :

Neglecting delayed neutrons, we put b = 0 in Eq. (2.35) and ignore Eq. (2.36), thus obtaining

$$\dot{X} = aX + aX^2 \tag{2.37}$$

where we have written X in place of X_1. This is a one-dimensional continuous system with one parameter and two fixed points X = 0 and X = -1.

2.5.3 Discrete Models :

The two direct feedback models presented in this section will also be studied in their discretized versions. We can discretize the continuous systems by taking differences in place of differentials. Discretizing Eq. (2.37), we get

$$X_{n+1} = (a\,\delta + 1)\,X_n + a\,\delta\,X_n^2 \tag{2.38}$$

where,

$\delta = \Delta\,\tau =$ step size of discretization, and

$X_n, X_{n+1} =$ n th and n+1 th iterates of the variable X, respectively.

This is a one-dimensional discrete dynamical system or map with one parameter.

Similarly, we can discretize the system of Eqs. (2.35) and (2.36) to obtain

$$X_{n+1} = (a\,\delta - b\delta + 1)\,X_n + b\,\delta\,Y_n + a\,\delta\,X_n^2 \tag{2.39}$$

$$Y_{n+1} = \delta\,X_n + (1 - \delta)\,Y_n \tag{2.40}$$

where, as before, $\delta = \Delta\,\tau$ is the step size. We have written X and Y for the variables X_1 and X_2 of the continuous system and subscripts denote the number of iterations. This is a two-dimentional discrete dynamical system or map with three parameters.

2.6. Values of the Parameters

The dynamical systems constructed in the previous sections involve some dimensionless parameters. These parameters have been defined in terms of the original physical parameters. The typical physical parameters for thermal and fast reactors are listed in Table 1. From the physical parameters, the dimensionless parameters may be calculated.

* * * *

CHAPTER 3

INVESTIGATIONS ON THE DYNAMICAL SYSTEMS FOR NUCLEAR FISSION REACTORS

In the previous chapter, we constructed dynamical systems ranging from one-dimensional to four-dimensional continuous and discrete systems. Now we proceed to investigate the behavior of these systems by analytical as well as numerical methods, with an eye on the possibility of the existence of limit cycles. The requisite concepts for analysis and computation have been taken from Refs. [14-27] and briefly mentioned in Appendix-A.

3.1. One-Temperature Feedback Models

First of all, we study the dynamical systems of Section 2.4.

3.1.1. Model without Delayed Neutrons :

This is a two-dimensional system of equations

$$\dot{X}_1 = aX_2 + aX_1X_2 \tag{3.1}$$

$$\dot{X}_2 = cX_1 - cX_2 \tag{3.2}$$

having two fixed points (0,0) and (-1, -1).

Local Behavior at the Operating Point (0,0)

The Jacobian matrix at this point is

$$\begin{bmatrix} 0 & a \\ c & -c \end{bmatrix}$$

whose characteristic equation is

$$\lambda^2 + c\lambda - ac = 0 \qquad (3.3)$$

having the roots (eigenvalues)

$$\lambda_{1,2} = -c/2 \pm (1/2)\sqrt{c^2 + 4ac}$$

From this, we can infer that

(1) This point is unstable for $a > 0$ (assuming $c > 0$) and locally stable for $a < 0$.

(2) From the movement of the eigenvalues in the complex-plane [Fig.1], we see that for $a > -c/4$ (and $c > 0$), λ_1 and λ_2 are real. At $a = -c/4$, they merge and then move vertically away from the real axis as 'a' decreases further. Therefore, locally (i.e. for small disturbances) the operating point behaves like an unstable node for $a \geq 0$, a stable node for $-c/4 < a < 0$, and a stable focus for $a < -c/4$ with oscillations becoming faster as 'a' decreases.

(3) To get pure imaginary eigenvalues, we must have $c^2 \leq 4ac$ and $c = 0$, which is not possible. Therefore, **Hopf bifurcation as well as existence of any periodic orbits in the neighborhood of the origin (i.e. the operating point) is ruled out.**

Local behavior at the Shutdown Point (-1, -1)

The Jacobian matrix at this point has the eigenvalues

$$\lambda_1 = -a \text{ and } \lambda_2 = -c.$$

Therefore,

(1) This point is locally stable for a > 0 and unstable for a < 0.

(2) Since the eigenvalues are real, no Hopf bifurcation is possible and periodic orbits in the neighborhood of the shutdown point are rule out.

Global Behavior

If this system has no limit cycles, the two fixed points constitute the global set of attractors and repellers. By combining the local behavior at the two fixed points, we can draw the global phase-portrait.

Numerical Experiments

To verify this global phase-portrait, we performed numerical experiments with a = -1.0 and c = 1.6 taking several initial conditions. The resulting plot is shown in Fig. 2. It confirms the existence of a stable focus at the operating point (0,0,0). For simulating the dynamical system, IMSL routine DVERK based on Verner's method was used with an interactive main program.

Some Further Remarks

Although we found no Hopf bifurcation in the system of Eqs. (3.1) to (3.2), we can create a Hopf bifurcation by slightly modifying the system as

$$\dot{X}_1 = aX_2 + aX_1X_2 \tag{3.1}$$

$$\dot{X}_2 = cX_1 + (d-c)X_2 \tag{3.4}$$

where d is a new parameter. d = 0 corresponds to the original system. The characteristic equation of the new system at (0,0) is

$$\lambda^2 - (d-c)\lambda - ac = 0 \tag{3.5}$$

The eigenvalues now are

$$\lambda_{1,2} = (d-c)/2 \pm (1/2)\sqrt{(d-c)^2 + 4ac}$$

which become purely imaginary at d = c, causing a Hopf bifurcation (assuming $a<0$ and $c>0$, which represents a realistic case). As d increases beyond c, the eigenvalues go from left-half to right-half of the complex-plane as shown in Fig.3. Therefore, from the Hopf bifurcation theorem, it follows that there exists either a stable limit cycle for $d>c$ or unstable limit cycle for $d<c$ or a nonlinear center at d = c. However as we will see in the next section, we find periodic solutions for the case d = c. **It is on this basis that periodic solutions in the system of Eqs. (3.1) - (3.2) are ruled out globally.**

3.1.2. Model without Delayed Neutrons and with Constant Power Removal

This model is given by the system of equations

$$\dot{X}_1 = aX_2 + aX_1X_2 \tag{3.6}$$

$$\dot{X}_2 = cX_1 \tag{3.7}$$

which has only one fixed point (0,0). We note that this system can also be obtained from the system of Eqs. (3.1) and (3.4) by setting d = c for which we indicated the possibility of a nonlinear center. At (0,0), this system has

the Jacobian matrix

$$\begin{bmatrix} 0 & a \\ c & 0 \end{bmatrix}$$

whose characteristic equation is

$$\lambda^2 - ac = 0 \tag{3.8}$$

and the eigenvalues are

$$\lambda_{1,2} = \pm \sqrt{ac}$$

which are purely imaginary for $a < 0$, $c > 0$. Therefore, the linear system has a center at (0,0) which may or may not be retained in the nonlinear system.

Numerical Experiments

To find out whether or not a nonlinear center exists, we performed numerical experiments with $a = -1$, $c = 1.6$ taking several initial conditions and each time found a closed curve around (0,0), thus indicating the existence of a nonlinear center [Fig.4]. For some initial conditions, the curve went below $X_2 = -1$ line which is unrealistic.

3.1.3. Model with Delayed Neutrons :

Now we come to a three-dimensional system of the equations

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + aX_1X_3 \tag{3.9}$$

$$\dot{X}_2 = X_1 - X_2 \tag{3.10}$$

$$\dot{X}_3 = cX_1 - cX_3 \tag{3.11}$$

having two fixed points (0,0,0) and (-1, -1, -1).

Local Behavior at the Operating Point (0,0,0)

The Jacobian matrix at this point is

$$\begin{bmatrix} -b & b & a \\ 1 & -1 & 0 \\ c & 0 & -c \end{bmatrix}$$

whose characteristic equation is

$$\lambda^3 + (c + b + 1)\lambda^2 + c(b + 1 - a)\lambda - ac = 0 \qquad (3.12)$$

For studying the movement of eigenvalues in the complex-plane, we rearrange the above equation as

$$\lambda(\lambda + c)(\lambda + b + 1) = ac(\lambda + 1) \qquad (3.13)$$

For the case a = 0 (no feedback), we find that all the three eigenvalues are real

$$\lambda_1 = 0, \quad \lambda_2 = -c, \quad \lambda_3 = -b - 1$$

For the other cases, $a < 0$ (negative feedback) and $a > 0$ (positive feedback) we can find the prohibited regions for the eigenvalues on the real line and hence investigate what happens as a decreases or increases starting from zero. We identify four cases :

(1) Case $a < 0$, $c > 1$ (assuming $b + 1 > c$) :

$$\lambda \notin (-\infty, -b-1] \cup [-c, -1] \cup [0, \infty) \qquad (3.14)$$

From this restriction and with the help of numerical experiments, we find that, as a decreases from a = 0, λ_1 tends to -1 ; whereas λ_2 and λ_3

approach each other, merge at a negative value and then move vertically away from the real axis [Fig.5a]. Therefore, the operating point behaves locally as a stable node or a stable focus.

(2) Case $a < 0$, $0 < c < 1$ (assuming $b > 0$) :

$$\lambda \notin (-\infty, -b-1] \cup [-1, -c] \cup [0, \infty) \tag{3.15}$$

From this constraint and with the help of numerical experiments, we find an interesting movement of eigenvalues in the complex-plane [Fig.5.b]. As a decreases from $a = 0$, λ_1 and λ_2 approach each other, merge, become complex conjugates, move to the left and land again on the real axis in the interval (-b-1, -1). Then one of the eigenvalues tends to -1, whereas the other merges with the third eigenvalue and forms complex conjugates which move away from the real axis vertically. Therefore, the operating point locally behaves as a stable node or a stable focus.

(3) Case $a > 0$, $c > 1$ (assuming $b+1 > c$) :

$$\lambda \notin [-b-1, -c] \cup [-1, 0] \tag{3.16}$$

From this prohibition we may conclue that no two eigenvalues are free to merge and form complex conjugates, and one of the eigenvalues is positive. Therefore, the operating point locally behaves like an unstable node.

(4) Case $a > 0$, $0 < c < 1$ (assuming $b > 0$) :

$$\lambda \notin [-b-1, -1] \cup [-c, 0] \tag{3.17}$$

From this restriction, we may conclude, as in the previous case, that no two eigenvalues are free to merge and form complex conjugates and one of the eigenvalues is positive. Therefore, the operating point locally behaves like an unstable node.

Local Behavior at the Shutdown Point (-1, -1, -1)

The Jacobian matrix at this point is

$$\begin{bmatrix} -b-a & b & 0 \\ 1 & -1 & 0 \\ c & 0 & -c \end{bmatrix}$$

whose characteristic equation is

$$(\lambda + c)[\lambda^2 + (a+b+1)\lambda + a] = 0 \qquad (3.18)$$

having the eigenvalues

$$\lambda_1 = -c,$$

$$\lambda_{2,3} = -(a+b+1)/2 \pm (1/2)\sqrt{(a+b+1)^2 - 4a}$$

$$= -(a+b+1)/2 \pm (1/2)\sqrt{(a+b-1)^2 + 4b}$$

From physical considerations, we must have $b > 0$. Therefore, all the eigenvalues are real and all of them are negative for $a > 0$, but not for $a < 0$. Therefore, locally, the shutdown point behaves like a stable node for $a > 0$ and an unstable node for $a < 0$.

Hopf Bifurcation at the Shutdown Point

Pure imaginary eigenvalues at (-1, -1, -1) occur at $b = -a-1$ for

$a > 0$ and the derivative

$$\frac{d}{db} \mathrm{Re}(\lambda_2) < 0$$

Therefore, for fixed $a > 0$, $c > 0$, this is a case of Hopf bifurcation in which a pair of complex conjugate eigenvalues crosses the imaginary exis from right to left as b increases beyond the critical value $b^+ = -a-1$ while the third eigenvalue remains in the left-half plane . From Hopf bifurcation theorem, it follows that **there exists either a stable limit cycle for $b > b^+$, or an unstable limit cycle for $b < b^+$ or a nonlinear center at $b = b^+$.** We are interested only in $b > 0$ for which **a stable limit cycle may or may not exist.** Even if it exists, it is likely to be unrealistic, because it would surround the shutdown point (-1, -1, -1), sometimes going below it.

Hopf Bifurcation at the Operating Point

To locate Hopf bifurcation at (0,0,0), we suppose Eq. (3.12) has the roots $\lambda_{1,2} = \alpha \pm i\beta$, $\lambda_3 = r$ with r, α, β real. These roots are related by

$$\lambda_1 + \lambda_2 + \lambda_3 = -(c+b+1)$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = c(b+1-a)$$

$$\lambda_1\lambda_2\lambda_3 = ac$$

That is,

$$2\alpha + r = -(c+b+1) \tag{3.19}$$

$$\alpha^2 + \beta^2 + 2\alpha r = c(b+1-a) \tag{3.20}$$

$$(\alpha^2 + \beta^2)\, r = ac \tag{3.21}$$

For pure imaginary eigenvalues, $\alpha^+ = 0$ and $\beta^{+2} > 0$, therefore

$$r^+ = -(c+b+1) \tag{3.22}$$

$$\beta^{+2} = c\,(b+1-a^+) \tag{3.23}$$

$$\beta^{+2} r^+ = a^+ c \tag{3.24}$$

Therefore,

$$c(b+1-a^+)\,(c+b+1) = a^+ c$$

or

$$a^+ = (b+1)\,(c+b+1)/(c+b) \tag{3.25}$$

This is the Hopf bifurcation point provided that $\beta^{+2} > 0$, i.e.

$$c(b+1-a^+) > 0$$

or

$$ca^+ < c(b+1)$$

or

$$c(b+1)\,(c+b+1)/(c+b) < c(b+1)$$

Assuming that $c > 0$ and $b > 0$, we must have

$$c+b+1 < c+b$$

i.e. $\quad 1 < 0$

which is impossible.

Therefore Hopf bifurcation as well as the existence of any periodic orbits in the neighborhood of the operating point is ruled out.

Global Behavior

For negative feedback ($a < 0$), no limit cycles are expected surrounding either of the fixed points. Therefore, we can combine the local behavior at the two fixed points to get the global phase-portrait. It will be a combination of a stable node or a stable focus at (0,0,0) and an unstable node at (-1,-1,1).

For positive feedback ($a > 0$), there is no limit cycle around (0,0,0), but there may be a stable limit cycle around (-1, -1, -1). Therefore, disturbances from the operating point may either become unbounded or asymptotically tend to the limit cycle.

Numerical Experiments

Numerical simulations were performed to verify the phase-portraits predicted above. The parameter values were taken for a thermal reactor (Chooz Sena PWR) : $a = -2614$, $b = 2143$, $c = 1.678$ taking several initial conditions. The resulting orbits are given in Table 2 and their projection on $X_1 - X_3$ plane is shown in Fig.6. **The operating point behaves like a globally stable node, attracting all orbits.**

3.1.4. Higher-Order Feedback Models :

The dynamical systems obtained from these models are the same as those for the corresponding linear feedback models but for an additional

term in the first equation. The inclusion of this additional nonlinear term, from which the linear part was already separated, does not affect the eigenvalues at the operating point (0,0,0) but may affect those at the shutdown point (-1, -1, -1). Therefore, the local behavior at (0,0,0) is not affected but that at (-1, -1, -1) may be affected. The Hopf bifurcation at (0,0,0) will occur at the same parameter values as for the linear feedback models but at (-1, -1, -1), the Hopf bifurcation point may change. Moreover, the direction of branching and the stability of limit cycles may change due to the inclusion of higher order feedback. These things may affect the global behavior.

3.2. Two-Temperature Feedback Models

Now we study the dynamical systems of Section 2.3.

3.2.1. Model without Delay Neutrons :

This is a three-dimensional system of equations

$$\dot{X}_1 = aX_3 + qX_4 + aX_1X_3 + qX_1X_4 \tag{3.26}$$

$$\dot{X}_2 = (k-h)\,X_1 - kX_2 + hX_3 \tag{3.27}$$

$$\dot{X}_3 = fX_2 - fX_3 \tag{3.28}$$

having two fixed points (0,0,0) and (-1, -1, -1).

Local Behavior at the Operating Point (0,0,0)

The Jacobian matrix at this point is

$$\begin{bmatrix} 0 & a & q \\ k-h & -k & h \\ 0 & f & -f \end{bmatrix}$$

whose characteristic equation is

$$\lambda^3 + (k+f)\ \lambda^2 + (f-a)(k-h)\lambda - (a+q)(k-h)\ f = 0 \qquad (3.29)$$

For the case a + q = 0, we get the eigenvalues

$$\lambda_1 = 0$$

$$\lambda_{2,3} = -(k+f)/2 \pm (1/2)\sqrt{(f-k)^2 + 4f + 4a(k-h)}$$

Since $f > 0$, $h > 0$ and $k-h > 0$, we have real eigenvalues for

$$a > -\frac{(f-k)^2 + 4hf}{4(k-h)} = -a^* \text{ (say)}$$

For $a < -a^*$, we have a complex conjugate pair of eigenvalues with negative real part, which move vertically toward the real axis as a increases, merge and then move along the real axis in opposite directions. One of them becomes positive for $a > f$. All the time, the eigenvalue λ_1 remains zero [Fig. 7].

Thus, we can say that for a+q = 0, the local behavior of the system around (0,0,0) cannot be determined by linear analysis only (since one of the eigenvalues is zero).

Hopf Bifurcation at Operating Point

Since $k > 0$ and $f > 0$, no pure imaginary eigenvalues are possible for the case a+q = 0. Therefore, there is no Hopf bifurcation in this case.

Suppose we fix a+q = G = a constant and define new parameters (constants) F = k+f > 0 and H = k-h > 0. Then Eq. (3.29) becomes

$$\lambda^3 + F\ \lambda^2 + H(f-a)\lambda - GHf = 0 \qquad (3.30)$$

whose roots $\lambda_1, \lambda_2, \lambda_3$ are related by

$$\lambda_1 + \lambda_2 + \lambda_3 = -F \tag{3.31}$$

$$\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1 = H(f-a) \tag{3.32}$$

$$\lambda_1 \lambda_2 \lambda_3 = GHf \tag{3.33}$$

In the vicinity of Hopf bifurcation point, we have a pair of complex conjugate eigenvalues, say $\lambda_{1,2} = \alpha \pm i\beta$, $\beta^2 > 0$ and a real eigenvalue, say $\lambda_3 = r$. Then we have

$$2\alpha + r = -F \tag{3.34}$$

$$\alpha^2 + \beta^2 + 2\alpha r = H(f-a) \tag{3.35}$$

$$(\alpha^2 + \beta^2) r = GHf \tag{3.36}$$

At $a = a^+$ (Hopf bifurcation point), we have $\alpha^+ = 0$, $\beta^{+2} = \omega^+ > 0$. Putting these in the above equations and eliminating r^+ and ω^+, we get

$$a^+ = \frac{f(G+F)}{F}$$

$$\omega^+ = -\frac{GHf}{F}$$

$$r^+ = -F$$

Since H, F and f are positive, we must keep $G < 0$ to have $\omega^+ > 0$. We also note that the real eigenvalue is negative at the bifurcation point.

To determine the direction of movement of the eigenvalues in the complex-plane, we differentiate Eqs. (3.34) - (3.36) with respect to a, and get

$$2\alpha' + r' = 0 \tag{3.37}$$

$$2\alpha\alpha' + 2\beta\beta' + 2\alpha r' + 2r\alpha' = -H \tag{3.38}$$

$$r(2\alpha\alpha' + 2\beta\beta') + (\alpha^2 + \beta^2) r' = 0 \tag{3.39}$$

Putting the values of α, β, r at $a = a^+$, and solving for α' we get

$$\alpha'(a^+) = \frac{HF}{2}\left(1 - \frac{GHf}{F^3}\right)^{-1} \tag{3.40}$$

which is positive for H, F, f >0 and G < 0. Therefore, the complex conjugate eigenvalues cross the imaginary axis from left to right and the real eigenvalue remains in the left-half plane as a increases beyond a^+. From Hopf bifurcation theorem **it follows that there exists either a stable limit cycle for $a > a^+$ or an unstable limit cycle for $a < a^+$ or a nonlinear center at $a = a^+$.** [Fig. 8].

Numerical Experiments

Numerical simulations were performed for G = -5, k = 2, h = 0.5, and f = 8 for which $a^+ = 4$. Trajectories were computed taking a = 4.5 and two initial conditions. The resulting orbits are given in Table 3,4 and projection on $X_1 - X_2$ plane is shown in Figs. 9,10. The orbit starting outside the limit cycle spiralled into it, and the one starting inside the limit cycle also spiralled out to it. The dimensionless times taken by these orbits to practically approach the limit cycle were 80 and 120 respectively. The period of the limit cycle was approximately 12. **This indicates the existence of a stable limit cycle for $a > a^+$.**

3.2.2. Model with Delayed Neutrons :

This is a four-dimensional dynamical system of the equations

$$\dot{X}_1 = -bX_1 + bX_2 + aX_3 + qX_4 + aX_1X_3 + qX_1X_4 \quad (3.41)$$

$$\dot{X}_2 = X_1 - X_2 \quad (3.42)$$

$$\dot{X}_3 = (k-h)\,X_1 - kX_3 + hX_4 \quad (3.43)$$

$$\dot{X}_4 = fX_3 - fX_4 \quad (3.44)$$

having two fixed points (0,0,0,0) and (-1, -1, -1, -1).

Characteristic Polynomial at the Operating Point (0,0,0,0)

The Jacobian matrix at this point is

$$\begin{bmatrix} -b & b & a & q \\ 1 & -1 & 0 & 0 \\ k-h & 0 & -k & h \\ 0 & 0 & f & -f \end{bmatrix}$$

whose characteristic polynomial is given by

$$P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \quad (3.45)$$

where ,

$$a_1 = 1 + b + k + f$$

$$a_2 = f + k\,(1+b) + (a+f)\,(k-h)$$

$$a_3 = a + f(1+b+a+q)$$

$$a_4 = (a+q)\,(k-h)\,f$$

Locating Hopf Bifurcation by Direct Decomposition Technique

In this technique, we decompose the characteristic polynomial near the Hopf bifurcation point as

$$P(\lambda) = (\lambda^2 + \omega)(\lambda^2 + p_1 \lambda + p_2) + A\lambda + B \qquad (3.46)$$

coefficients in this equation are given recursively by

$$p_{-1} = 0 ; \; p_o = 1 \; ;$$

$$p_k = a_k - \omega p_{k-2} , \qquad k = 1, 2, \ldots , n-2 ;$$

$$A = a_{n-1} - \omega p_{n-3} ; \; B = a_n - \omega p_{n-2}$$

In our case, $n = 4$, and

$$p_{-1} = 0 ; \; p_o = 1$$

$$p_1 = a_1 - \omega p_{-1} = a_1 \qquad (3.47)$$

$$p_2 = a_2 - \omega p_o = a_2 - \omega \qquad (3.48)$$

$$A = a_3 - \omega p_1 = a_3 - \omega a_1 \qquad (3.49)$$

$$B = a_4 - \omega p_2 = a_4 - \omega (a_2 - \omega)$$

$$= a_4 - a_2 \omega + \omega^2 \qquad (3.50)$$

At the Hopf bifurcation point, the parameters should be such that $A = 0$, $B = 0$, i.e.

$$a_3 - \omega a_1 = 0 \qquad (3.51)$$

$$a_4 - a_2 \omega + \omega^2 = 0 \qquad (3.52)$$

From Eq. (3.51) we get

$$\omega = a_3/a_1 \tag{3.53}$$

Substituting the value of ω in Eq. (3.52) from Eq. (3.53), we get

$$a_4 - (a_3/a_1)(a_2 - a_3/a_1) = 0$$

or,

$$a_1^2 a_4 - a_1 a_2 a_3 + a_3^2 = 0$$

or,

$$a_1 a_2 a_3 = a_1^2 a_4 + a_3^2 \tag{3.54}$$

Moreover, for pure imaginary roots, we must have $\omega > 0$

i.e. ,

$$a_3/a_1 > 0 \tag{3.55}$$

Eqs. (3.54) and (3.55) are the necessary conditions for Hopf bifurcation in the system of Eqs. (3.41) - (3.44).

Thus, we see that there is a possibility of Hopf bifurcation at $a = a^+$. Periodic orbits around the operating point, therefore, cannot be ruled out in the vicinity of the operating point. Further analysis, however, appears to be too involved.

3.2.3. Higher-Order Feedback Models :

In the two-temperature feedback also, nonlinear feedback models may be considered. Here too, the effect of inclusion of higher-order feedback

will be similar to that in the one-temperature feedback case. The location of Hopf bifurcation point will remain unchanged but the direction of branching and stability of the limit cycles may change. Further bifurcations from limit cycle may also be affected by the extra nonlinear terms.

3.3. Direct Feedback Models

Now we study the simplest models i.e. those of Section 2.4.

3.3.1. Model without Delayed Neutrons :

This is the simplest of the models, as given by the equation :

$$\dot{X} = aX + aX^2 \tag{3.56}$$

having two fixed points $X = 0$ and $X = -1$. Its Jacobian is

$$dX/dX = a \quad \text{at} \quad X = 0$$

$$dX/dX = -a \quad \text{at} \quad X = -1$$

Therefore, $X = 0$ is unstable for $a > 0$ and locally stable for $a < 0$. Except for unrealistic initial conditions, $X < -1$, it is also globally asymptotically stable for $a < 0$. At $a = 0$, an exchange of stability occurs between the two fixed points.

Since this systems is a one-dimensional continuous dynamical system, it cannot have any limit cycles or periodic orbits.

3.3.2. Model with Delayed Neutrons :

This is a two-dimensional system of equations

$$\dot{X}_1 = (a-b)\,X_1 + bX_2 + aX_1^2 \tag{3.57}$$

$$\dot{X}_2 = X_1 - X_2 \tag{3.58}$$

having two fixed points (0,0) and (-1, -1) .

At the operating point (0,0) the Jacobian matrix is

$$\begin{bmatrix} a-b & b \\ 1 & -1 \end{bmatrix}$$

whose characteristic equation is

$$\lambda^2 + (b-a-1)\,\lambda - a = 0 \tag{3.59}$$

having the eigenvalues

$$\lambda_{1,2} = -(1/2)(b-a+1) \pm (1/2)\sqrt{(b-a+1)^2 + 4a}$$

$$= -(1/2)(b-a+1) \pm (1/2)\sqrt{(b-a-1)^2 + 4b}$$

For $b > 0$, both the eigenvalues are real. In addition, if $a < 0$, they are negative, hence a stable node. Keeping $b > 0$, as a increases past $a = 0$, one eigenvalue becomes positive. Therefore, there is a saddle-node bifurcation at $a = 0$.

Hopf Bifurcation at the Operating Point

Pure imaginary eigenvalues occur at $b = a-1$ for a fixed $a < 0$. As b increases beyond this value, the eigenvalues cross the imaginary axis from right to left. Therefore, from Hopf bifurcation theorem there exists either a stable limit cycle for $b < 0$ or an unstable limit cycle for $b > 0$ or a non-linear center at $b = 0$. We are interested in $b > 0$ for which **an unstable limit cycle is possible.**

3.3.3. Discrete Models

The discrete model without delayed neutrons is the one-dimensional map

$$X_{n+1} = (a\,\delta + 1)\,X_n + a\,\delta\,X_n^2 \tag{3.60}$$

which can be transformed to the well known logistic map

$$x_{n+1} = \mu\,x_n\,(1 - x_n) \tag{3.61}$$

by the transformation

$$x = \left(\frac{-a\,\delta}{a\,\delta + 1}\right) X$$

where $\mu = a\,\delta + 1.$

The logistic map is known to have chaotic behavior for $\mu \geq 4$. This corresponds to $a\,\delta \geq 3$, which means that a must be positive, and the dimensionless time step, δ, should be of the order 1 (because a is of the order 1), which is unrealistic discretization.

The discrete model with delayed neutrons is the two-dimensional map

$$X_{n+1} = (a\delta - b\delta + 1) X_n + b\delta\, Y_n + a\delta\, X_n^2 \qquad (3.62)$$

$$Y_{n+1} = \delta X_n + (1 - \delta)\, Y_n \qquad (3.63)$$

which can be transformed to the Henon map (also chaotic), taking $\delta = 1$,

$$x_{n+1} = 1 + y_n + \sigma x_n^2 \qquad (3.64)$$

$$y_{n+1} = b\, x_n \qquad (3.65)$$

by the transformation

$$x = AX - B,$$

$$y = b\,(AY - B)$$

where

$$A = a/\sigma \;,\; B = 2/(b-1),$$

$$\sigma = (1/4)\,(b-1)\,(b-a-1)$$

Here we had to take the dimensionless time step, δ, equal to unity, which is an unrealistic discretization.

Thus, we see that in both the discrete models discussed above, we had to take unrealistic discretizations to see chaos. The discrete models based on discrete population dynamics may exhibit different kind of behavior.

* * * *

CHAPTER 4

SOME ADDITIONAL INVESTIGATIONS

In addition to analysing the fission reactor dynamical systems for periodic behavior, we have, during the course of this work, also attempted to see if these systems, have any chaotic behavior. Methods for identifying chaos were studied. The method of Lyapunov exponents was applied to the two-temperature feedback model without delayed neutrons.

4.1. Methods for Identifying Chaos

In the last decade, new phenomena have been observed in all areas of nonlinear dynamics, principal among these being chaotic oscillations. Chaotic oscillations are the emergence of bounded, aperiodic random like motions from completely deterministic systems. Such motions had been known in fluid mechanics, but they have recently been observed in low-order mechanical and electrical systems and even in simple one-degree-of-freedom problems. The random character of the motions in these deterministic systems arises from sensitive dependence on initial conditions. This leads to exponential divergence of nearby trajectories.

Methods for identifying chaos include Lyapunov exponents, Poincare' map, Fourier spectrum, and the identification of routes to chaos through bifurcations. Details of these methods may be found in Refs.[21-24, 28-30].

4.1.1. Lyapunov Exponents :

Numbers that measure the exponential attraction or separation in time of two adjacent trajectories in phase space with different initial conditions are called Lyapunov exponents.

If one imagines a set of initial conditions within a sphere of radius ϵ in phase space, then for chaotic motions, trajectories originating in the sphere will map the sphere into an ellipsoid whose major axis grows exponentially with an exponent $\lambda > 0$, which is known as a Lyapunov exponent. If d_o is a measure of distance between two starting points, at a small but later time

$$d(t) = d_o 2^{\lambda t} \qquad (4.1)$$

The divergence of chaotic orbits can only be locally exponential, since if the system is bounded, as most physical experiments are, d (t) cannot go to infinity. Thus, to define a measure of this divergence of orbits, we must average the exponential growth at many points along a trajectory.

4.1.2. Poincare' Sections :

A Poincare' section is the sequence of points in phase-space generated by the penetration of a continuous evolution trajectory through a generalized surface or plane in the space. For a 3-dimensional case, we can take a plane, S, and place the points of intersection corresponding to a given direction of crossing. The plane is chosen so that the trajectory Γ intersects S at P_o, P_1,... ,

where the dynamics of the system are assumed to be such that the trajectory continually crosses from one side of S to the other. S can be any plane, but an appropriate choice yields sections that are more easily analysed. In a three-dimensional phase space, the Poincare' section of a periodic orbit is a single point or a set of a finite number of points. For a queasiperiodic orbit, it is a continuous closed curve, but is not traversed continuously by successive intersection points of the trajectory with the plane. For an aperiodic orbit, the Poincare' section is a set of points distributed along an unclosed curve. It can be reduced further to a one-dimensional first return map by defining a coordinate for each point on the curve and plotting each point of intersection against the previous one.

4.1.3. Fourier Transform and Power Sepctrum :

The Fourier transform of a discrete time series x_j is defined to be the operation creating a corresponding discrete series x_k such that

$$x_k = \frac{1}{\sqrt{n}} \sum_{j=1}^{n} x_j \exp\left(-i \frac{2\pi jk}{n}\right), \quad k = 1, \ldots, n. \qquad (4.2)$$

The power spectrum is the graph of S_k as a function of frequency f ($f=k.\Delta f$) where S_k is defined by

$$S_k = |x_k|^2 = \sum_{m=1}^{n} \psi_m \cos\left(\frac{2\pi mk}{n}\right) \qquad (4.3)$$

where ψ_m is the autocorrelation function of the signal x_j defined by

$$\psi_m = \frac{1}{n} \sum_{j=1}^{n} x_j x_{j+m} \qquad (4.4)$$

For periodic signals (with period T) we get a peak at the frequency 1/T, its sidelobes, and possibly a certain number of other peaks (and their sidelobes) that are harmonics of the fundamental frequency. For a quasiperiodic signal (with fundamental frequencies f_1 and f_2) several peaks at independent frequencies, $f = m_1 f_1 + m_2 f_2$, where m_1 and m_2 are positive integers. If f_1/f_2 is irrational, then the power spectrum is dense. For an aperiodic signal, the power spectrum is continuous. But this is not a conclusive evidence of an aperiodic or nonperiodic signal, because the power spectrum of a quasiperiodic signal with very high number of frequencies has a similar appearance.

4.2. Calculation of the Largest Lyapunov Exponent

The signs of the Lyapunov exponents provide a qualitative picture of a system's dynamics. In a three-dimensional continuous dissipative system the only possible spectra, and the attractors they describe, are as follows : (+, 0, -), a strnage attractor (chaos) ; (0, 0, -), a two-torus (quasiperiodic); (0, -, -), a limit cycle; and (-, -, -), a fixed point. Since the largest Lyapunov exponent dominates the behavior of a dynamical system, its calculation is important to determine the nature of the system. If it turns out to be positive, it confirms that the system exhibits chaos.

Suppose we want to calculate the largest Lyapunov exponent of a flow :

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \tag{4.5}$$

whose variational equations are :

$$\dot{\delta x} = A\,\delta x \tag{4.6}$$

where A is the Jacobian matrix. To calculate the largest positive Lyapunov exponent, we have to measure the divergence of nearby trajectories averaged over a long time. Eq. (4.6) is to be integrated along a trajectory over a small enough time interval starting with arbitrary initial conditions. Redefining the initial conditions, the process is to be continued along the same trajectory. Taking a sufficiently large number, N, of such steps, the largest Lyapunov exponent may be calculated as

$$\lambda = \frac{1}{t_N - t_o} \sum_{k=1}^{N} \log_2 \frac{d(t_k)}{d(t_{k-1})} \tag{4.7}$$

The methods for calculating the complete spectrum of Lyapunov exponents of a system are discussed in Refs. [28-30].

4.2.1. Algorithm and Program :

Based on the method give above, an algorithm for calculating the largest Lyapunov exponent was written which is as follows :

1. Starting from a point in the state-space, calculate the trajectory over a sufficient time to allow the orbit to approach the attractor.
2. Choose an arbitrary variation vector of unit magnitude.
3. Starting from the present point in the state-space, integrate the equations of the flow and its variation simultaneously, over a suitably chosen time step.

4. Calculate the magnitude, d (t_k), of the variation vector, where k is the present step number.

5. Calculate $\log_2$ d (t_k) and add it to the sum of all the previous values of $\log_2$ d(t_i), i = 1, ..., k.

6. Divide the new sum by the time over which the integration has been done to obtain the latest estimate of the largest Lyapunov exponent.

7. Normalize the variation vector to unit magnitude.

8. Repeat steps 3 to 7 until a satisfactory estimate of largest Lyapunov exponent is obtained.

An interactive program [Appendix-B] based on the above algorithm was written in FORTRAN IV and implemented on DEC 1090 computer. The program was tested for the Lorenz and Rossler systems (both chaotic), and the results agreed with those in Refs. [28, 29].

The calculation of the complete spectrum of the Lyapunove exponents is described in Refs. [28-30]. The program of Wolf et al [28] was made interactive and implemented on DEC 1090 computer for the Lorenz and Rossler systems. The results agreed with those of the author.

4.2.2. Calculation of the largest Lyapunov Exponent for the Two-Temperature Feedback Model without Delayed Neutrons :

For this reactor model, Hopf bifurcation had been shown analytically in Section 3.2.1. and the existence of a stable limit cycle observed by numerical

Lyapunov exponent for this system taking two sets of values of parameters. The parameter values were chosen such that the operating point would become unstable and Hopf bifurcation theorem would predict the existence of a stable limit cycle.

1. **Parameter Set No. 1**

G = -2, k =2, h = 1.6, f = 10.0 for which Hopf bifurcation occurs at $a^+ = 8\frac{1}{3}$; a = 9 was chosen.

First Attempt

Execution was done for 80,000 time steps of size 10^{-3} skipping the first 1000 steps and printing every 100th step. The tolerance chosen to be 10^{-6}. The execution on DEC 1090 took 5.24s of CPU time.

The results of this attempt are shown in Table . Towards the end of the execution, the estimate of the Lyapunov exponent was seen to be oscillating between positive values only (+0.02 and +0.10). It was suspected that, for these parameter values, further bifurcations of the limit cycle had taken place, and the existence of a chaotic attractor was suspected. To confirm this, another attempt with a longer time of evolution was made.

Second Attempt

This time, 6000 steps of size 1 were taken, skipping the first 100 steps and printing every 100th step. The tolerance was chosen as 10^{-5}. The execution took 2 min. 11.24s of CPU time on DEC 1090.

Towards the end of the execution, the estimate of the largest Lyapunov exponent was seen to be oscillating between small positive and negative values, of the order 10^{-4}, which can be taken to be approaching zero. Hence, the existence of a chaotic orbit is ruled out and that of a limit cycle or a torus is expected. This demonstrates that, taking insufficient time of evolution may lead to erroneous conclusions.

2. **Parameter Set No.2**

G = -5, h = 0.5, k = 2, f = 8 for which Hopf bifurcation occurs at $a^+ = 4$; a = 4.5 was chosen.

For this parameter set, numerical experiments had already been performed and reported in Section 3.2.1, where a stable limit cycle had been observed. The execution for calculating the largest Lyapunov exponent was done for 4000 time steps of size 1, skipping first 100 steps and printing every 100th step. As for the previous parameter set, here also the estimate was seen to be oscillating between small positive and negative values of the order 10^{-4}, which can be taken to be approaching zero. Hence, the existence of a chaotic orbit is ruled out in this case also, and a limit cycle or a torus is expected. This, in conjunction with the results of numerical simulations, may be used to infer that a stable limit cycle exists for this set of parameter values.

CHAPTER 5

CONCLUSION

In the preceeding chapters nuclear fission reactors were modelled using point reactor kinetics and reactivity feedback effects. Transforming the equations to dimensionless form, dynamical systems for various reactor models were constructed. These dynamical systems were then investigated for the possibility of the existence of limit cycles by analytical methods as well as numerical experiments. In one of the models (the two-temperature feedback model without delayed neutrons) a stable limit cycle was predicted using the method of Hopf bifurcation. This was confirmed by numerical experiments. Some additional investigations on the possibility of chaos were also done, by computing the largest Lyapunov exponent. The result was in the negative for the parameter values chosen (after the Hopf bifurcation).

5.1. Summary of the Dynamic Modeling

The dynamic models were obtained using the equations of point reactor kinetics and energy balances. The feedback effects of the energy balance equations were included in the point kinetic equations.

The dynamic equations thus obtained were transformed in such a way that all the state variables, the independent variable, and the parameters became dimensionless, while still allowing direct physical interpretation. The

state variables were normalized with respect to the equilibrium operating point of the final system (after the autonomous disturbances) and this point was translated to the origin. The time was normalized with respect to the decay constant of delayed neutron precursors. Thus, the number of parameters in the system was greatly reduced. The shutdown point was also made independent of all the parameters by choosing the normalizations appropriately.

5.2. Summary of the Investigations

The dynamical systems constructed for various reactor models were investigated by analytical methods as well as numerical experiments. The movement of eigenvalues was studied with the aim of locating a Hopf bifurcation. The direct decomposition technique was used to locate Hopf bifurcation in the two-temperature feedback model with delayed neutrons. Numerical experiments were performed to verify the theoretical predictions. The results are as follows :

1. One-Temperature Feedback Models

   a) without delayed neutrons : Hopf bifurcation was ruled out. A stable focus at the origin was predicted and experimentally confirmed.

   b) without delayed neutrons and with constant power removal : Possibility of a nonlinear center was indicated and experimentally confirmed.

   c) with delayed neutrons : Hopf bifurcation was ruled out. A stable node at the origin was predicted and experimentally confirmed.

d) higher-order feedback models : Compared with the linear feedback models.

2. Two-Temperature Feedback Models

a) without delayed neutrons. Hopf bifurcation was located and periodic orbits were predicted. Numerical experiments indicated the occurrence of a supercritical Hopf bifurcation and the existence of a stable limit cycle.

b) with delayed neutrons : The necessary condition for the Hopf bifurcation was determined by using the direct decomposition technique.

c) higher-order feedback models : Compared with the linear feedback models.

3. Direct Feedback Models

a) without delayed neutrons : One-dimensional system, hence no limit cycles are possible.

b) with delayed neutrons : Hopf bifurcation was located and the existence of an unstable limit cycle was predicted.

c) discrete models : the above two models were discretized and reduced to the logistic map and the Henon map respectively, both of which are well known for exhibiting chaos. However, it was noted that, in our systems, chaos can occur only for unrealistic discretization.

4. For detecting chaos, in the continuous systems, a program for computing the largest Lyapunov exponent was written and tested. The computations for the two-temperature feedback model without delayed neutrons yielded a zero value of the largest Lyapunov exponent, thus ruling out the possibility of chaos for the parameter values chosen, and further confirming the existence of a stable limit cycle.

5.3. Suggestions

For further investigations in the direction of the present work, the following suggestions can be given :

1) The direction of branching, the stability of the limit cycles and their domains of attraction need to be investigated further.

2) Absence of Hopf bifurcation does not exclude periodic orbits of large amplitude and periods. This needs further investigation.

3) The possibility of chaos for realistic values of the parameters may be investigated by computing the largest Lyapunov exponent. If one had access to a supercomputer, one could calculate the largest Lyapunov exponent for many values of the parameters and thus determine the ranges of parameter values for which chaos exists.

4) The nature of the attractor may be studied by taking a Poincare section.

5) The behavior of the dynamical systems may also be investigated

by taking the Fourier transform and studying the power spectrum.

5.4 Epilogue

In the present work, several mathematical models based on different physical assumptions have been constructed and studied, with the objective of locating limit cycles. Different models exhibited different kind of behavior, ranging from globally stable fixed points to centers and limit cycles. Further investigations on mathematical models may be carried out, but it is important to note that they are based on certain idealizations and approximations. Predictions based on mathematical models need to be verified by experiments on the actual physical systems.

* * * *

APPENDIX - A

SOME MATHEMATICAL CONCEPTS

A.1. The Hopf Bifurcation Theorem

Let us consider a system of ordinary differential equations

$$\frac{dx}{dt} = F(x, \mu) \tag{A.1}$$

which has a stationary point $x = 0 \in R^n$ and the critical value of the bifurcation parameter μ is 0. Then if

(1) $F(0, \mu) = 0$ for μ in an open interval containing 0, and $0 \in \mathbb{R}^n$ is an isolated stationary point of F,

(2) F is analytic in x and μ in a neighborhood of (0,0) in $\mathbb{R}^n \times \mathbb{R}^1$,

(3) $A(\mu) = D_x F(0, \mu)$ has a pair of complex conjugate eigenvalues λ and $\bar{\lambda}$ such that

$$\lambda(\mu) = \alpha(\mu) + i\omega(\mu), \tag{A.2}$$

where
$$\omega(0) = \omega_o > 0, \quad \alpha(0) = 0, \quad \alpha'(0) \neq 0, \tag{A.3}$$

(4) The remaining n-2 eigenvalues of A (0) have strictly negative real parts,

then the system (A.1) has a family of periodic solutions : there is an $\varepsilon_H > 0$ and an analytic function

$$\mu^H(\varepsilon) = \Sigma_2^{\infty} \mu_i^H \varepsilon^i \qquad (0 < \varepsilon < \varepsilon_H) \qquad (A.4)$$

such that for each $\varepsilon \in (0, \varepsilon_H)$ there exists a periodic solution $p_\varepsilon(t)$ occurring for $\mu = \mu^H(\varepsilon)$. If $\mu^H(\varepsilon)$ is not identically zero, the first non-vanishing coefficient μ_i^H has an even subscript, and there is an $\varepsilon_1 \in (0, \varepsilon_H]$ such that $\mu^H(\varepsilon)$ is either strictly positive or strictly negative for $\varepsilon \in (0, \varepsilon_1)$. For each $L > 2\pi/\omega_o$ there is a neighborhood η of $x = 0$ and an open interval I containing 0 such that for any $\mu \in I$ the only nonconstant periodic solutions of (A.1) with periods less than L which lie in η are members of the family $p_\varepsilon(t)$ for values of ε satisfying $\mu^H(\varepsilon) = \mu, \varepsilon \in (0, \varepsilon_H)$. The period $T^H(\varepsilon)$ of $p_\varepsilon(t)$ is an analytic function

$$T^H(\varepsilon) = (2\pi/\omega_o)[1 + \Sigma_2^{\infty} \tau_i^H \varepsilon^i] \qquad (0 < \varepsilon < \varepsilon_H) \qquad (A.5)$$

Exactly two of the Floquet exponents of $p_\varepsilon(t)$ approach 0 as $\varepsilon \to 0$. One is 0 for $\varepsilon \in (0, \varepsilon_H)$, and the other is an analytic function

$$\beta^H(\varepsilon) = \Sigma_2^{\infty} \beta_i^H \varepsilon^i \qquad (A.6)$$

The periodic solution $p_\varepsilon(t)$ is orbitally asymptotically stable with asymptotic phase if $\beta^H(\varepsilon) < 0$ but is unstable if $\beta^H(\varepsilon) > 0$.

4.2 Numerical Techniques Used

Numerical experiments on a dynamical system may be used to investigate its global behavior and to locate limit cycles, if any. This can be done by numerically simulating the dynamical system on a digital computer. This

differential equations over a large number of steps.

Standard library subroutines such as DVERK (in IMSL library) and D02BAF and D02EAF (in NAG library) are available for solving a system of ordinary differential equations. In general, Runge-Kutta methods, such as Verner's method (DVERK) and Merson's method (D02BAF) can be used without problem. However, for stiff systems, these methods fail, and methods specially suited for stiff systems, such as Gear's method (a predictor-corrector method) have to be used. The subroutines D02EAF and D02EBF in NAG library are based on this method and were used for integrating the systems based on models with delayed neutrons.

APPENDIX - B

AN INTERACTIVE PROGRAM FOR COMPUTING THE LARGEST LYAPUNOV EXPONENT

```
*****************************************************************

This is an interactive program for computing the largest
Lyapunov exponent(ELAMDA) of a continuous dynamical system
of dimension N .

The equations defining a dynamical system and its variational
equations are integrated simultaneously over a given step size
starting with given initial conditions and an arbitrarily chosen
variation vector of unit magnitude. After a step of integration,
the variation vector is normalizedto unity and the process is
repeated. The largest Lyapunov exponent is computed by calculating
the ratio of the logarithm (to the base 2) of the ratio of the
magnitude of the variation vector after and before integration,
averaged over a large number of steps.
*****************************************************************

      PROGRAM LYAP
      PARAMETER N=3,NN=6,NW=9

N = # of equations in the system ;
NN = 2*N = total # of equations to be integrated.

      EXTERNAL LORENZ,ROSLER,REACTR

Subroutine containing the equations to be integrated is to be
declared in the above EXTERNAL statement.

      INTEGER NEQ,IW,IFAIL,I,J,K,ID,NSKIP,NSTEP

NSTEP = # of steps of integration ;
NSKIP = # of steps to be skipped initially ;
ID    = input/output rate of steps.

      REAL X,XEND,TOL,STEP,ZNORM1,ZNORM2,TERM,SUM,ELAMDA

X = the independent variable ;
ZNORM1 = norm of the position vector ;
ZNORM2 = norm of the variation vector ;
ELAMDA = estimate of the largest Lyapunov exponent .

      REAL Y(NN),W(NN,NW),CC(24)
```

```
C     Y = array containing the N dependent variables and
C         their N variations .
C
          REAL A,B,C,D,E,F
          COMMON A,B,C,D,E,F
C
C     Values of the parameters
C
          DATA C,D,F,G/2.0,0.5,8.0,-5.0/
          DATA C,D,F,G/2.0,1.6,10.0,-2.0/
          TYPE 6
6         FORMAT(1H ,'P1. give A')
          ACCEPT*,A
          E=G-A
          OPEN(UNIT=23,DEVICE='DSK',FILE='LYAP.DAT')
C
C...Initial conditions for nonlinear system
C
          TYPE 5
5         FORMAT(1H ,'P1. give the initial conditions ')
          ACCEPT*, (Y(I),I=1,N)
C
C...Initial conditions for linear system (arbitrarily chosen
C...variation vector of unit norm)
C
          Y(4)=0.75
          Y(5)=0.5
          Y(6)=SQRT(3./16.)
C
C...Integration tolerance,time per step,# of integration steps,
C...and I/O rate
C
          TYPE 1
1         FORMAT(1H ,'P1. give TOL,STEP,NSTEP,ID,NSKIP')
          ACCEPT*,TOL,STEP,NSTEP,ID,NSKIP
C
C...Initialization for integrator
C
          NEQ=NN;IW=NW
          X=0.0;XX=X+FLOAT(NSKIP)*STEP
          SUM=0.0
          INDEX=0
100       DO 200 I=1,NSTEP
              INDEX=INDEX+1
              IND=1
              XEND=X+STEP
C
C...Integrate the ODE's by an IMSL routine using Verner's method
C
              CALL DVERK(NEQ,REACTR,X,Y,XEND,TOL,IND,CC,NEQ,W,IER)
```

```
            IF(IND.NE.3.OR.IER.NE.0) TYPE 2,IND,IER
2       FORMAT(1H ,'Error check: IND=',I1,3X,'IER=',I3)
C
C...Calculate the norms of the position and variation vectors
C
            ZNORM1=0.0;ZNORM2=0.0
            DO 20 J=1,N
                ZNORM1=ZNORM1+Y(J)**2
                ZNORM2=ZNORM2+Y(N+J)**2
20          CONTINUE
            ZNORM1=SQRT(ZNORM1)
            ZNORM2=SQRT(ZNORM2)
C
C...Normalize the variation  vector
C
            DO 30 K=N+1,NN
                Y(K)=Y(K)/ZNORM2
30          CONTINUE
            IF(INDEX.LE.NSKIP) GOTO 50
C
C...calculate the largest(positive) Lyapunov exponent
C
            TERM=ALOG(ZNORM2)/ALOG(2.)
            SUM=SUM+TERM
            ELAMDA=SUM/(X-XX)
C
C...Print every IO iterations
C
50          IF(MOD(INDEX,IO).NE.0) GOTO 200
                WRITE(23,3) X,ELAMDA,ZNORM1,ZNORM2,Y
                TYPE *,X,ELAMDA
3       FORMAT(1H ,10(2X,E11.4))
200     CONTINUE
C               TYPE 3,X,ELAMDA,ZNORM1,ZNORM2,Y
        TYPE 4
4       FORMAT(1H ,'How many more steps ?')
        ACCEPT*,NSTEP
        IF(NSTEP.GT.0) GOTO 100
        END
C*************************************************************************
        SUBROUTINE LORENZ(N,X,Y,YDOT)
C
C   The Lorenz system (chaotic) to test the program .
C
        REAL X,Y(6),YDOT(6)
        YDOT(1)=16.*(Y(2)-Y(1))
        YDOT(2)=-Y(1)*Y(3)+45.92*Y(1)-Y(2)
        YDOT(3)=Y(1)*Y(2)-4.*Y(3)
        YDOT(4)=16.*(Y(5)-Y(4))
        YDOT(5)=(45.92-Y(3))*Y(4)-Y(5)-Y(1)*Y(6)
```

```
      YDOT(6)=Y(2)*Y(4)+Y(1)*Y(5)-4.*Y(6)
      RETURN
      END
C*************************************************************************
      SUBROUTINE ROSLER(N,X,Y,YDOT)
CCC
CCC   The Rossler system (chaotic) to test the program .
      PARAMETER A=0.15,B=0.2,C=10.0
      REAL X,Y(6),YDOT(6)
      YDOT(1)=-Y(2)-Y(3)
      YDOT(2)=Y(1)+A*Y(2)
      YDOT(3)=B+Y(3)*(Y(1)-C)
      YDOT(4)=-Y(5)-Y(6)
      YDOT(5)=Y(4)+A*Y(5)
      YDOT(6)=Y(3)*Y(4)+(Y(1)-C)*Y(6)
      RETURN
      END
C*************************************************************************
      SUBROUTINE REACTR(N,X,Y,YDOT)
CCC
CCC   Two temperature feedback model without delayed nutrons
      INTEGER N
      REAL X,Y(3),YDOT(3)
      REAL A,B,C,D,E,F
      COMMON A,B,C,D,E,F
CCC
CCC   The dynamical system
      YDOT(1)=A*Y(2)+E*Y(3)+A*Y(1)*Y(2)+E*Y(1)*Y(3)
      YDOT(2)=(C-D)*Y(1)-C*Y(2)+D*Y(3)
      YDOT(3)=F*Y(2)-F*Y(3)
CCC
CCC   The variational equations
      YDOT(4)=(A*Y(2)+E*Y(3))*Y(4)+(A+A*Y(1))*Y(5)+(E+E*Y(1))*Y(6)
      YDOT(5)=(C-D)*Y(4)-C*Y(5)+D*Y(6)
      YDOT(6)=F*Y(5)-F*Y(6)
      RETURN
      END
```

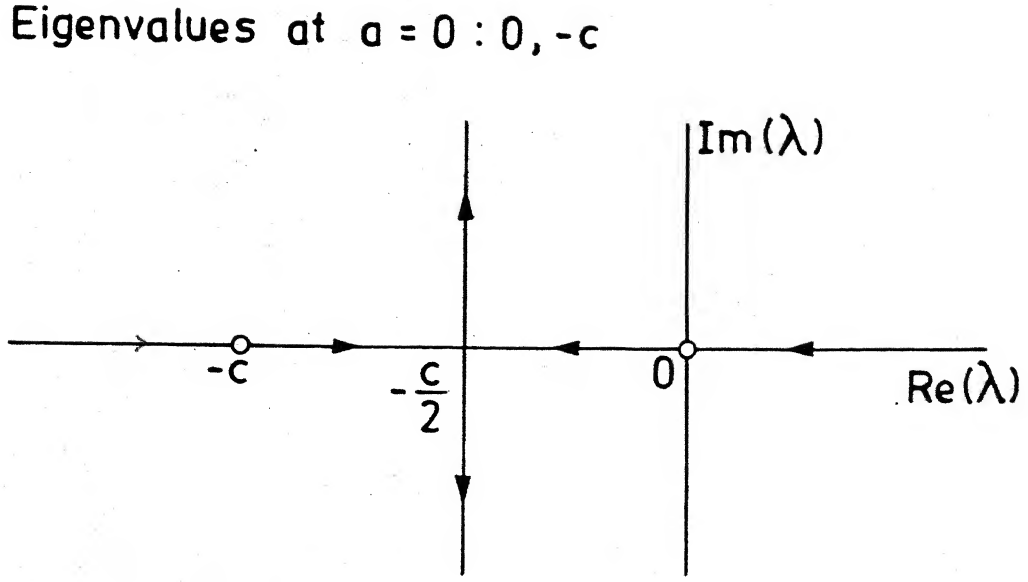


Fig. 1. Movement of eigenvalues in one-temperature model without delayed neutrons as a decreases ($c>0$).

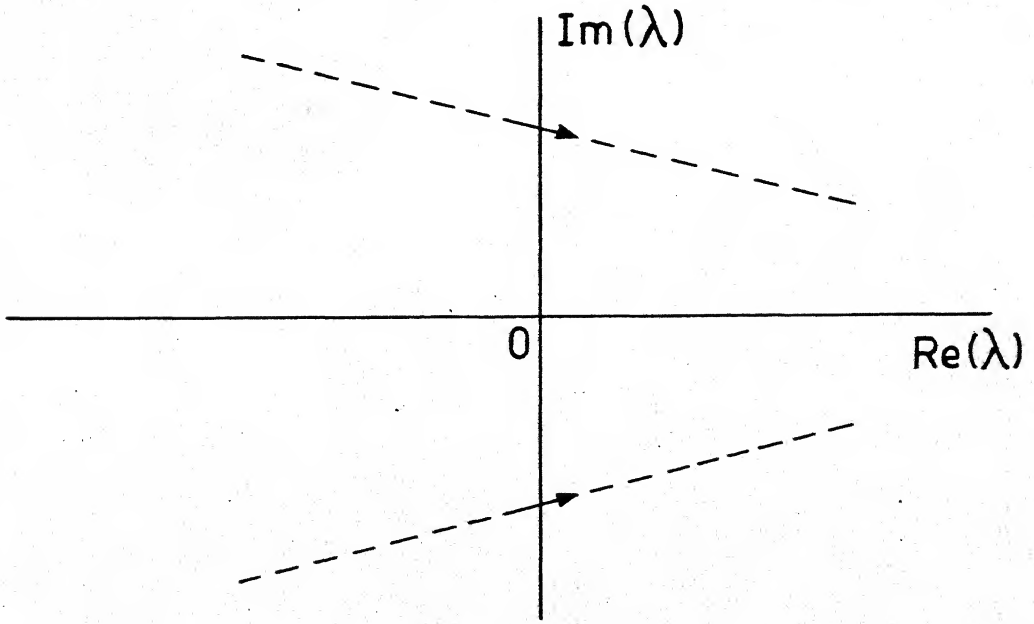


Fig. 3. Hopf bifurcation created in one-temperature model without delayed neutrons by an extra parameter.

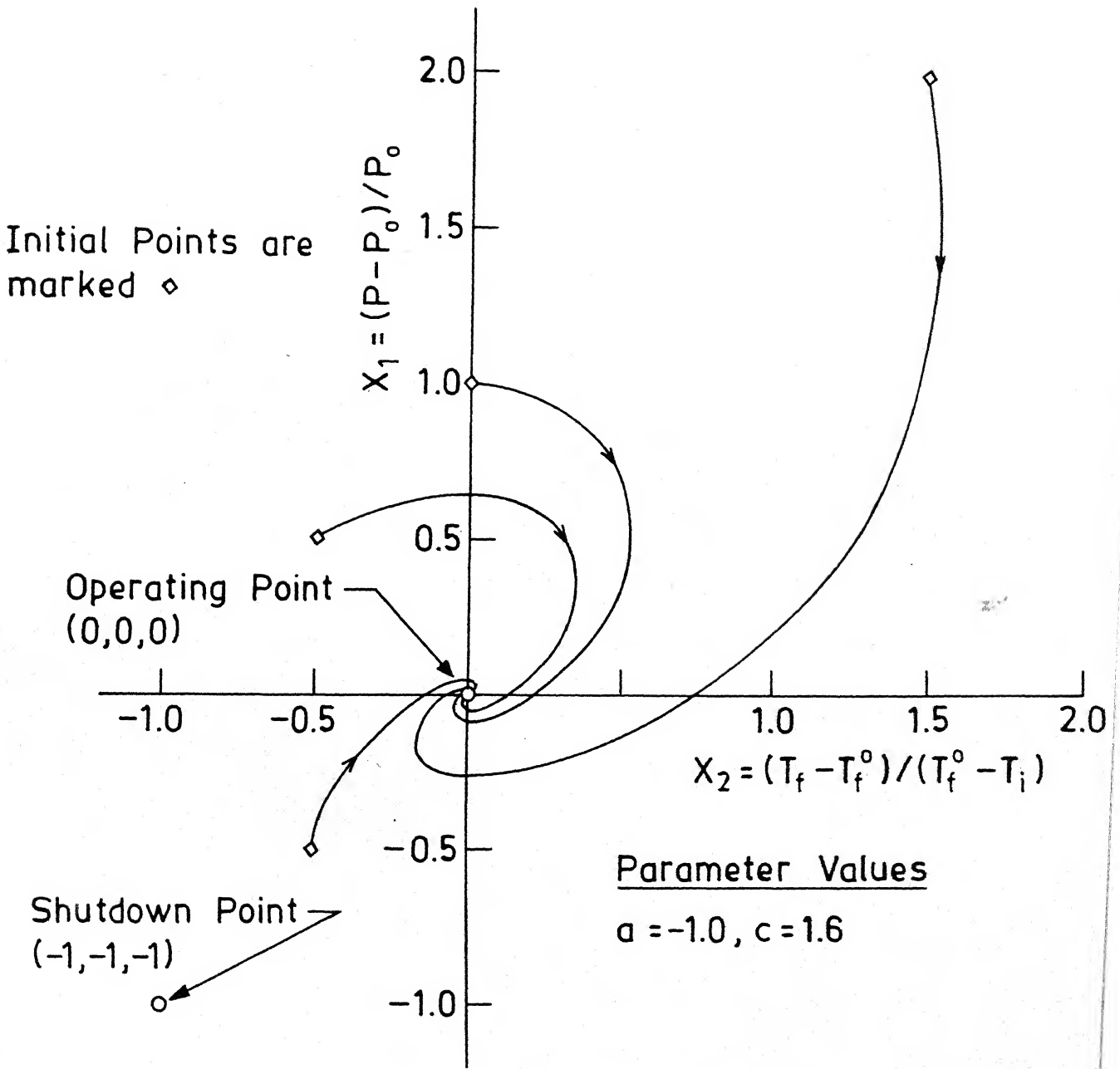


Fig. 2. Orbits for One-Temperature Feedback Model without Delayed Neutrons.

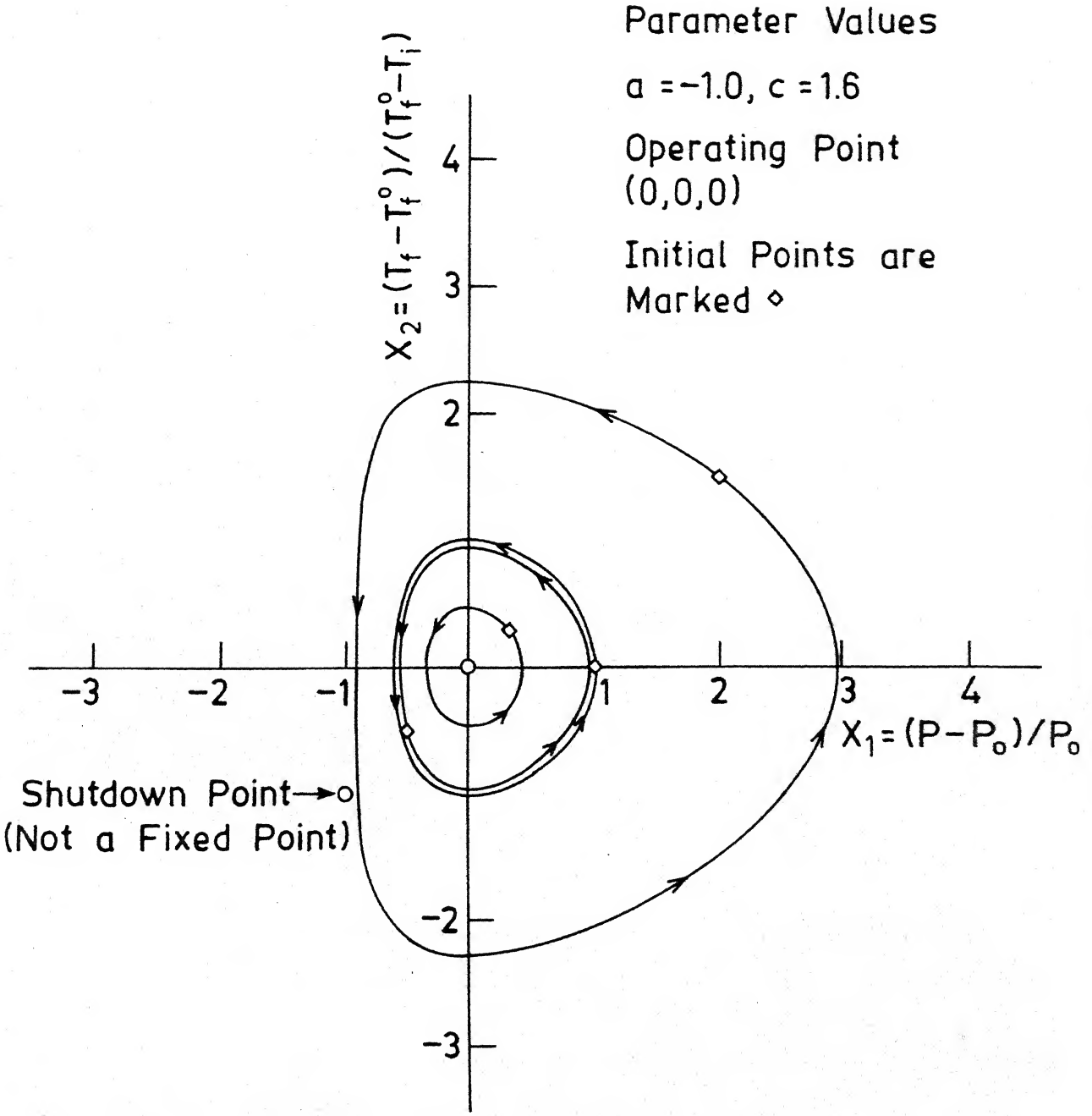
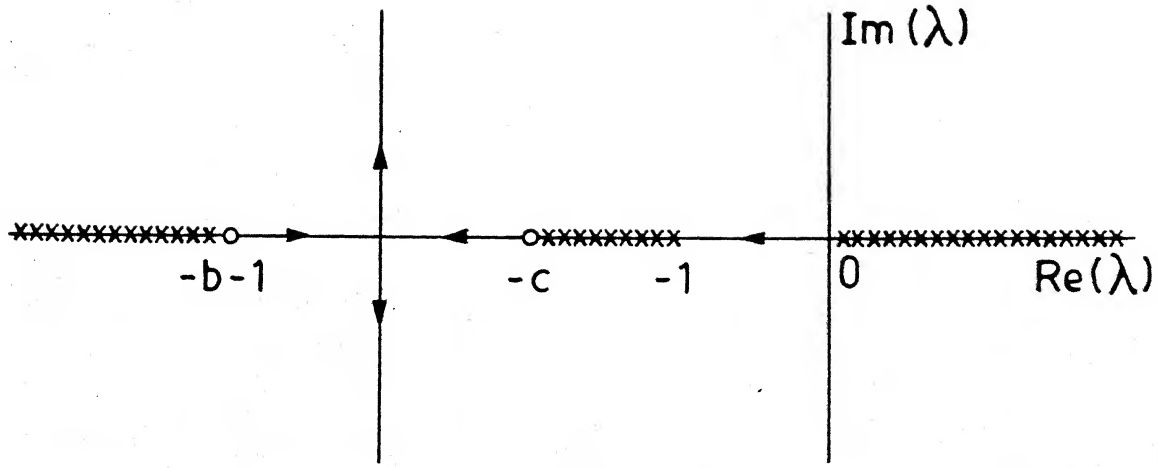


Fig. 4. Orbits for One-Temperature Feedback Model without Delayed Neutrons and with Constant Power Removal.

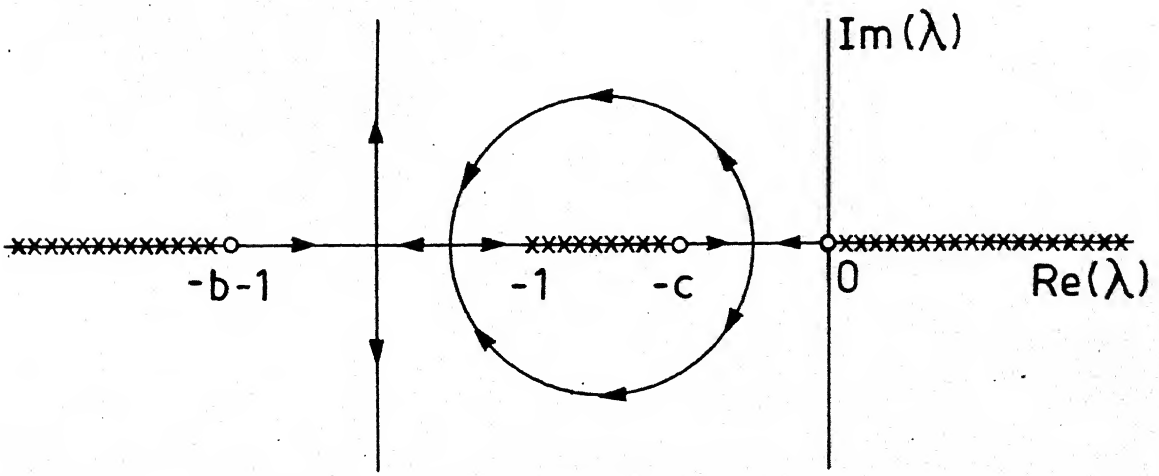
Notations :

o Eigenvalues at a = 0

***** Prohibited regions



(a) Case a<0, c>1



(b) Case a<0, 0<c<1

Fig. 5. Movement of eigenvalues in one-temperature model with delayed neutrons as a decreases.

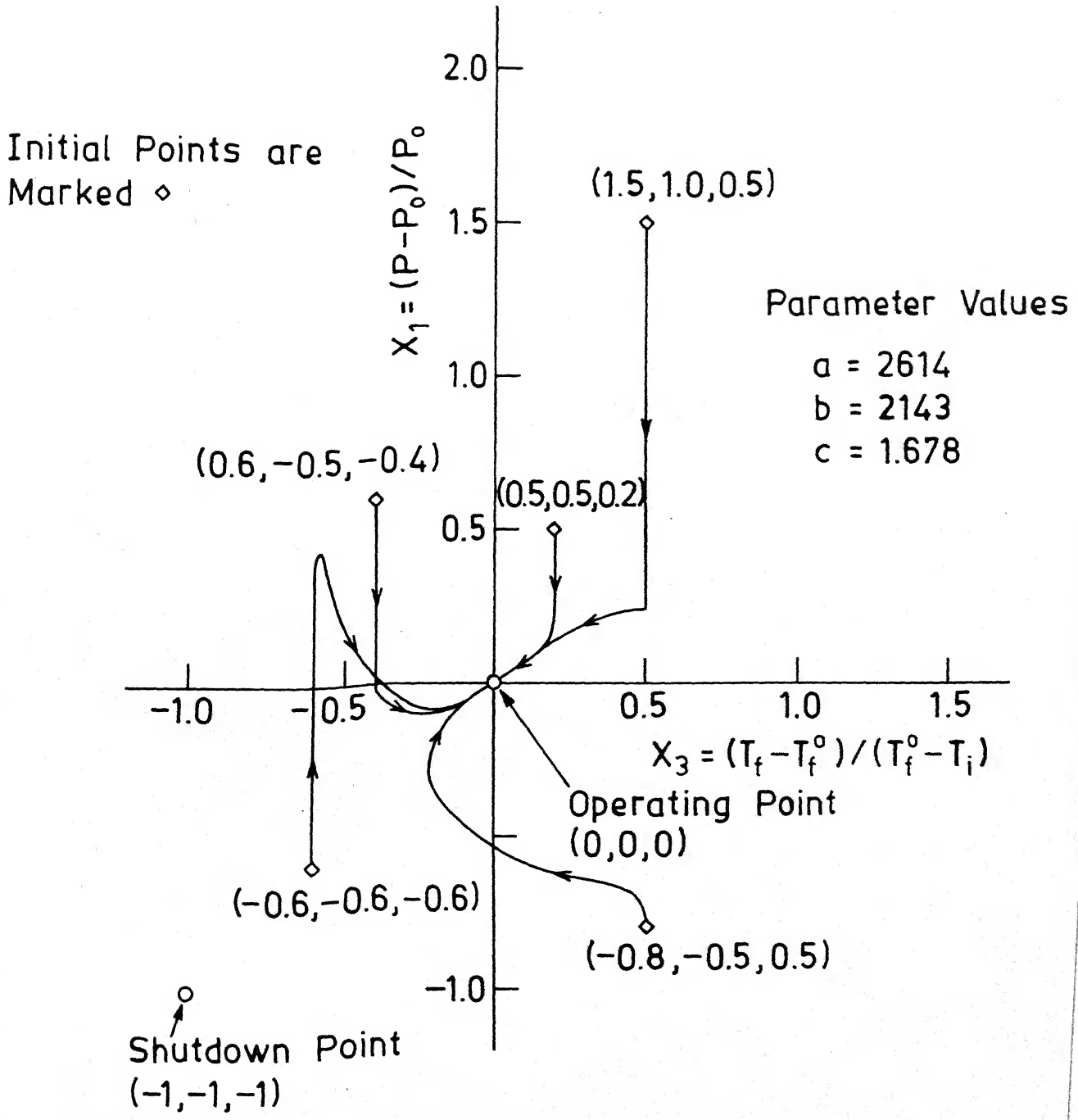


Fig. 6. Projection of Orbits for One-Temperature Feedback Model with Delayed Neutrons.

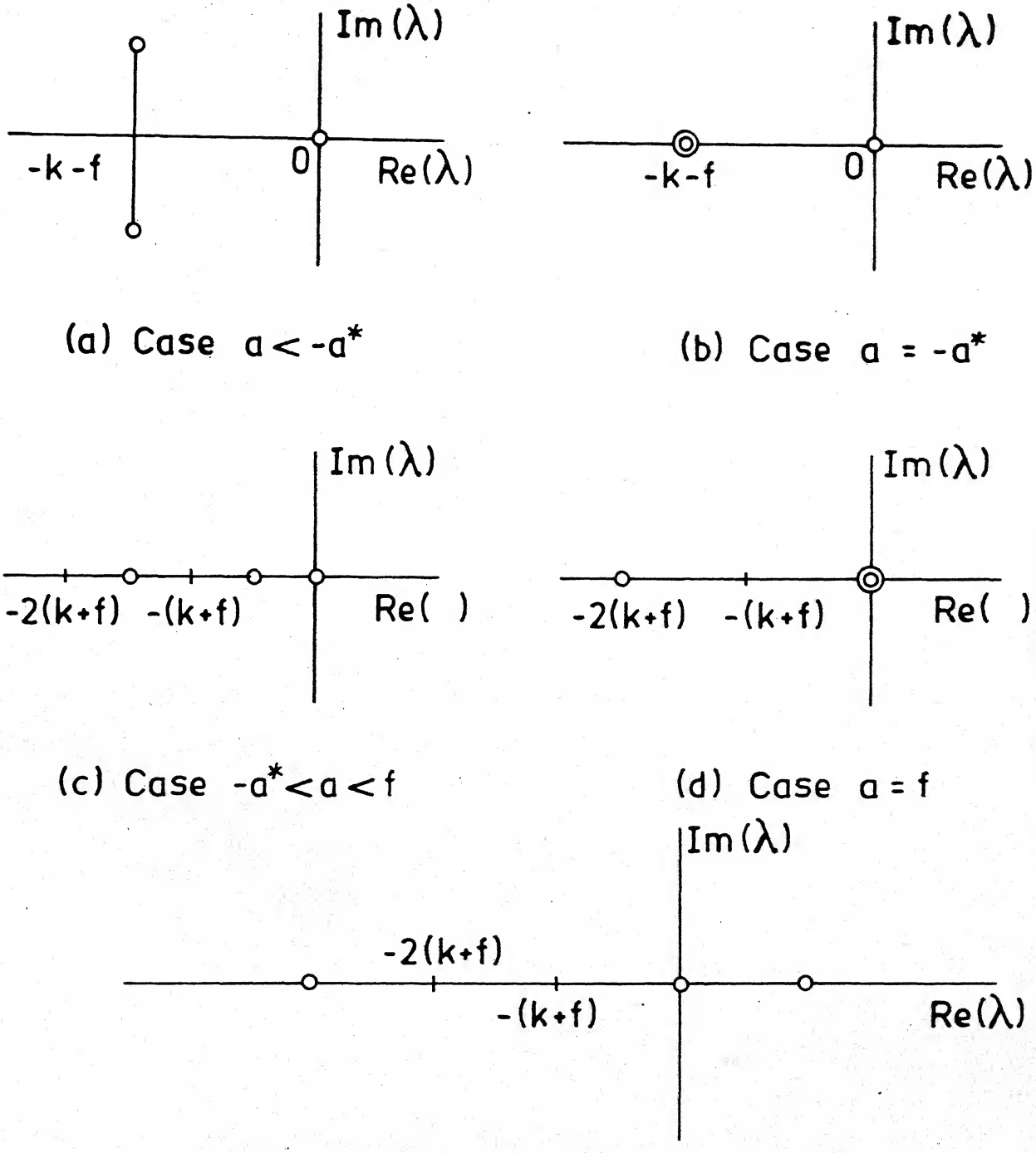


Fig. 7. Movement of eigenvalues in two-temperature model without delayed neutrons for the case $a+q=0$.

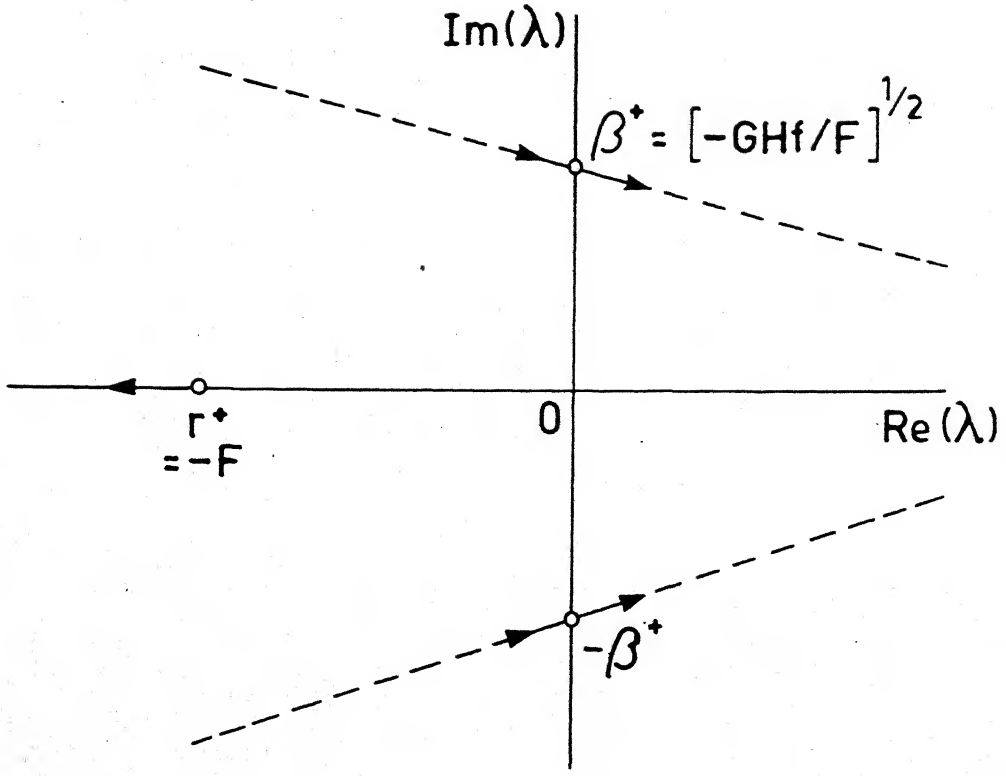


Fig. 8. HOPF bifurcation in two-temperature model without delayed neutrons as the parameter a increases past the critical value a^*.

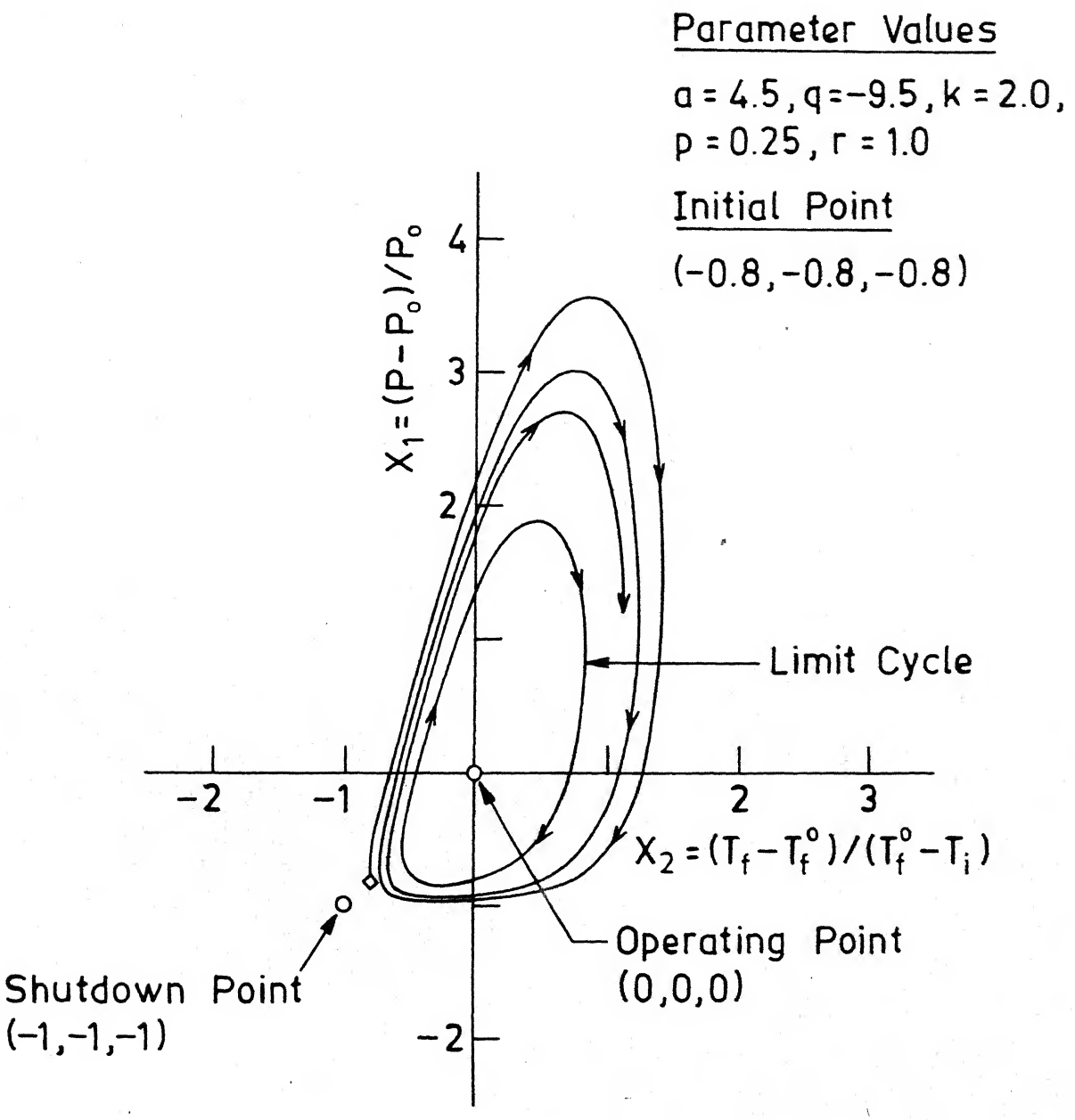


Fig. 9. Projection of an Orbit for Two-Temperature Feedback Model without Delayed Neutrons.

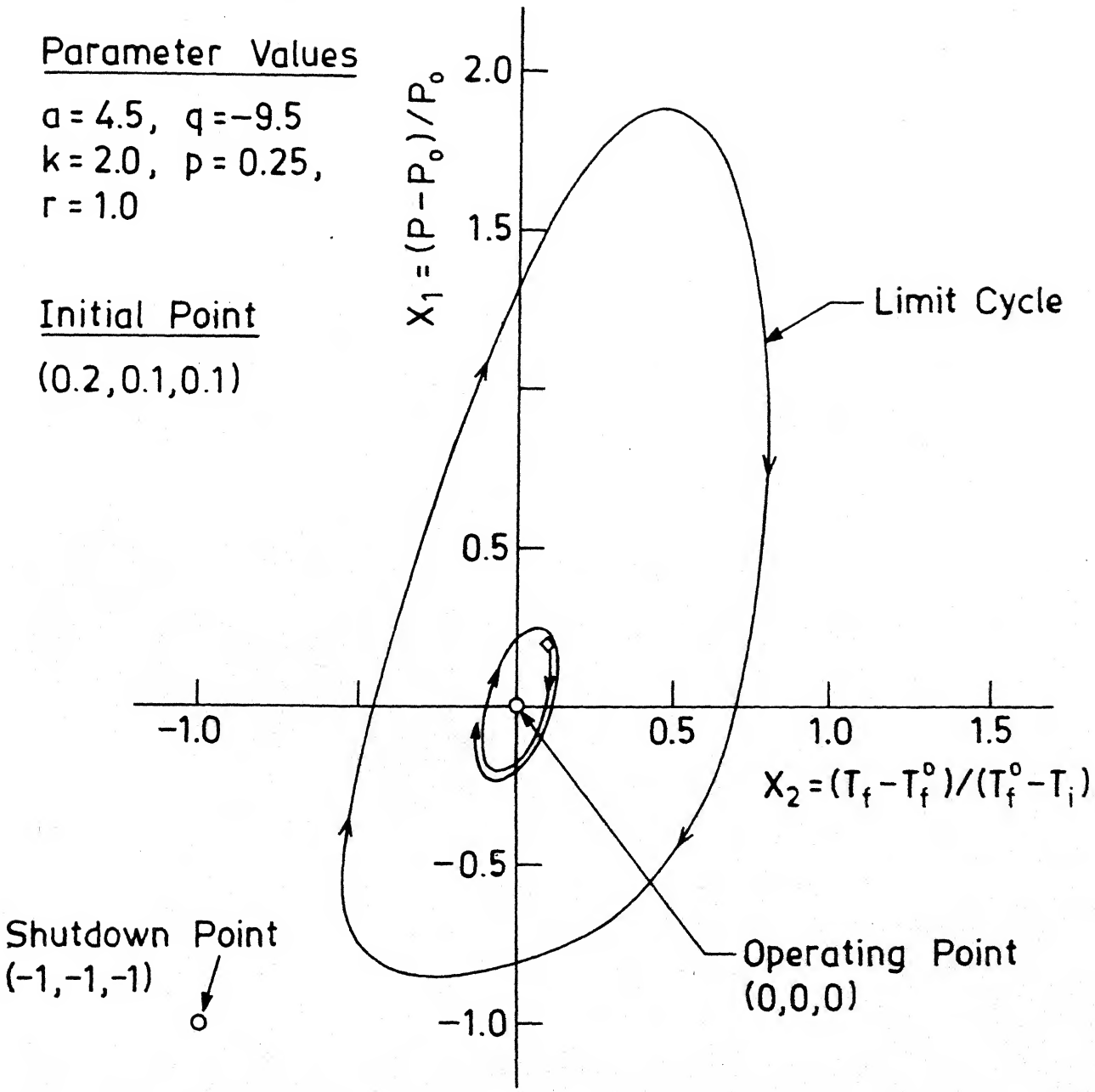


Fig. 10. Projection of Another Orbit for Two-Temperature Feedback Model without Delayed Neutrons.

TABLE 1

TYPICAL VALUES OF PARAMETERS

PARAMETER	VALUE	
	THERMAL REACTOR	FAST REACTOR
β	0.0066	0.0035
λ	$0.077\ s^{-1}$	$0.065\ s^{-1}$
l	$4.0 \times 10^{-5} s$	$5.0 \times 10^{-7} s$
P_o	1040 MW	3000 MW
α_f , α_c	-10^{-5} to $+ 10^{-5}$	-10^{-5} to $+ 10^{-5}$
K	$10^{-7}\ KJ^{-1}$	$10^{-7}\ KJ^{-1}$
T_f^o	1343 K	1783 K
T_c^o	500 K	600 K
T_i	600 K	700 K

In models without delayed neutrons, we take l to be of the order of the effective lifetime, i.e.,

$$l \cong \beta/\lambda$$

TABLE 2. TRAJECTORIES CALCULATED FOR ONE-TEMPERATURE MODEL WITH DELAYED NEUTRONS

TOL= 0.10E-02

INITIAL CONDITIONS:

0.6000E+00 0.5000E+00 0.2000E+00

X	Y(1)	Y(2)	Y(3)
.0000E+00	0.6000E+00	0.5000E+00	0.2000E+00
.1000E-01	0.2034E+00	0.4972E+00	0.2002E+00
.2000E-01	0.2010E+00	0.4942E+00	0.2003E+00
.3000E-01	0.1987E+00	0.4913E+00	0.2002E+00
.4000E-01	0.1954E+00	0.4884E+00	0.2002E+00
.5000E-01	0.1941E+00	0.4855E+00	0.2001E+00
.6000E-01	0.1919E+00	0.4826E+00	0.2000E+00
.7000E-01	0.1898E+00	0.4797E+00	0.1998E+00
.8000E-01	0.1875E+00	0.4768E+00	0.1997E+00
.9000E-01	0.1856E+00	0.4739E+00	0.1994E+00
.1000E+00	0.1835E+00	0.4710E+00	0.1992E+00
.1000E+00	0.1835E+00	0.4710E+00	0.1992E+00
.2000E+00	0.1652E+00	0.4428E+00	0.1953E+00
.3000E+00	0.1499E+00	0.4156E+00	0.1895E+00
.4000E+00	0.1358E+00	0.3897E+00	0.1824E+00
.5000E+00	0.1257E+00	0.3650E+00	0.1744E+00
.6000E+00	0.1150E+00	0.3417E+00	0.1659E+00
.7000E+00	0.1073E+00	0.3198E+00	0.1573E+00
.8000E+00	0.9958E-01	0.2991E+00	0.1488E+00
.9000E+00	0.9251E-01	0.2798E+00	0.1405E+00
.1000E+01	0.8603E-01	0.2616E+00	0.1326E+00
.1000E+01	0.8603E-01	0.2616E+00	0.1326E+00
.2000E+01	0.4355E-01	0.1334E+00	0.7059E-01
.3000E+01	0.2210E-01	0.6831E-01	0.3681E-01
.4000E+01	0.1153E-01	0.3536E-01	0.1922E-01
.5000E+01	0.6075E-02	0.1840E-01	0.1005E-01
.6000E+01	0.3154E-02	0.9573E-02	0.5241E-02
.7000E+01	0.1688E-02	0.5107E-02	0.2800E-02
.8000E+01	0.9242E-03	0.2798E-02	0.1534E-02
.9000E+01	0.5102E-03	0.1545E-02	0.8477E-03
.1000E+02	0.2720E-03	0.8236E-03	0.4522E-03
.1000E+02	0.2720E-03	0.8236E-03	0.4522E-03
.2000E+02	0.7147E-05	0.2161E-04	0.1188E-04
.3000E+02	0.1351E-05	0.4093E-05	0.2246E-05
.4000E+02	0.8319E-06	0.2514E-05	0.1383E-05
.5000E+02	0.8431E-06	0.9442E-06	0.5199E-06
.6000E+02	0.6525E-07	0.1950E-06	0.1078E-06
.7000E+02	0.4034E-07	0.1205E-06	0.6662E-07
.8000E+02	0.1543E-07	0.4601E-07	0.2546E-07
.9000E+02	0.2597E-08	0.7688E-08	0.4274E-08
.1000E+03	0.3257E-09	0.9959E-09	0.5487E-09

TABLE 2. (CONTD.)

TOL= 0.10E-02

INITIAL CONDITIONS:

0.1500E+01 0.1000E+01 0.5000E+00

X	Y(1)	Y(2)	Y(3)
.0000E+00	0.1500E+01	0.1000E+01	0.5000E+00
.1000E-01	0.2414E+00	0.9928E+00	0.4963E+00
.2000E-01	0.2407E+00	0.9853E+00	0.4921E+00
.3000E-01	0.2400E+00	0.9779E+00	0.4879E+00
.4000E-01	0.2393E+00	0.9705E+00	0.4837E+00
.5000E-01	0.2386E+00	0.9633E+00	0.4797E+00
.6000E-01	0.2379E+00	0.9561E+00	0.4756E+00
.7000E-01	0.2372E+00	0.9489E+00	0.4717E+00
.8000E-01	0.2364E+00	0.9418E+00	0.4678E+00
.9000E-01	0.2357E+00	0.9348E+00	0.4639E+00
.1000E+00	0.2349E+00	0.9279E+00	0.4601E+00

.1000E+00	0.2349E+00	0.9279E+00	0.4601E+00
.2000E+00	0.2254E+00	0.8615E+00	0.4246E+00
.3000E+00	0.2159E+00	0.8006E+00	0.3932E+00
.4000E+00	0.2070E+00	0.7445E+00	0.3652E+00
.5000E+00	0.1987E+00	0.6929E+00	0.3400E+00
.6000E+00	0.1853E+00	0.6453E+00	0.3173E+00
.7000E+00	0.1750E+00	0.6013E+00	0.2965E+00
.8000E+00	0.1651E+00	0.5605E+00	0.2773E+00
.9000E+00	0.1565E+00	0.5226E+00	0.2596E+00
.1000E+01	0.1474E+00	0.4874E+00	0.2430E+00

.1000E+01	0.1474E+00	0.4874E+00	0.2430E+00
.2000E+01	0.7803E-01	0.2453E+00	0.1272E+00
.3000E+01	0.4055E-01	0.1250E+00	0.6652E-01
.4000E+01	0.2105E-01	0.6422E-01	0.3466E-01
.5000E+01	0.1088E-01	0.3310E-01	0.1801E-01
.6000E+01	0.5673E-02	0.1721E-01	0.9399E-02
.7000E+01	0.3007E-02	0.9103E-02	0.4982E-02
.8000E+01	0.1586E-02	0.4810E-02	0.2637E-02
.9000E+01	0.8433E-03	0.2559E-02	0.1404E-02
.1000E+02	0.4667E-03	0.1412E-02	0.7748E-03

.1000E+02	0.4667E-03	0.1412E-02	0.7748E-03
.2000E+02	0.9767E-05	0.2955E-04	0.1624E-04
.3000E+02	0.2719E-05	0.8225E-05	0.4522E-05
.4000E+02	0.3744E-06	0.1125E-05	0.6206E-06
.5000E+02	0.1597E-06	0.4798E-06	0.2645E-06
.6000E+02	0.2542E-07	0.7599E-07	0.4202E-07
.7000E+02	0.8101E-08	0.2422E-07	0.1340E-07
.8000E+02	0.1971E-08	0.5901E-08	0.3266E-08
.9000E+02	0.4053E-09	0.3155E-08	0.1746E-08
.1000E+03	0.1315E-09	0.4091E-09	0.2252E-09

TABLE 2. (CONTD.)

TOL= 0.10E-02

INITIAL CONDITIONS:

-0.8000E+00 -0.5000E+00 0.5000E+00

X	Y(1)	Y(2)	Y(3)
.0000E+00	-0.8000E+00	-0.5000E+00	0.5000E+00
.1000E-01	-0.6850E+00	-0.5019E+00	0.4802E+00
.2000E-01	-0.6824E+00	-0.5037E+00	0.4609E+00
.3000E-01	-0.6788E+00	-0.5055E+00	0.4419E+00
.4000E-01	-0.6751E+00	-0.5072E+00	0.4233E+00
.5000E-01	-0.6714E+00	-0.5088E+00	0.4050E+00
.6000E-01	-0.6676E+00	-0.5104E+00	0.3871E+00
.7000E-01	-0.6637E+00	-0.5120E+00	0.3696E+00
.8000E-01	-0.6599E+00	-0.5135E+00	0.3524E+00
.9000E-01	-0.6559E+00	-0.5149E+00	0.3356E+00
.1000E+00	-0.6520E+00	-0.5163E+00	0.3191E+00
.1000E+00	-0.6520E+00	-0.5163E+00	0.3191E+00
.2000E+00	-0.6095E+00	-0.5272E+00	0.1724E+00
.3000E+00	-0.5525E+00	-0.5328E+00	0.6531E-01
.4000E+00	-0.5120E+00	-0.5332E+00	-0.3602E-01
.5000E+00	-0.4595E+00	-0.5237E+00	-0.1053E+00
.6000E+00	-0.4070E+00	-0.5196E+00	-0.1557E+00
.7000E+00	-0.3573E+00	-0.5065E+00	-0.1905E+00
.8000E+00	-0.3118E+00	-0.4901E+00	-0.2125E+00
.9000E+00	-0.2719E+00	-0.4712E+00	-0.2245E+00
.1000E+01	-0.2379E+00	-0.4506E+00	-0.2289E+00
.1000E+01	-0.2379E+00	-0.4506E+00	-0.2289E+00
.2000E+01	-0.8873E-01	-0.2493E+00	-0.1445E+00
.3000E+01	-0.4334E-01	-0.1289E+00	-0.7328E-01
.4000E+01	-0.2219E-01	-0.6654E-01	-0.3719E-01
.5000E+01	-0.1139E-01	-0.3440E-01	-0.1909E-01
.6000E+01	-0.5888E-02	-0.1786E-01	-0.9874E-02
.7000E+01	-0.3097E-02	-0.9384E-02	-0.5174E-02
.8000E+01	-0.1627E-02	-0.4925E-02	-0.2711E-02
.9000E+01	-0.8728E-03	-0.2645E-02	-0.1456E-02
.1000E+02	-0.4704E-03	-0.1427E-02	-0.7847E-03
.1000E+02	-0.4704E-03	-0.1427E-02	-0.7847E-03
.2000E+02	-0.9752E-05	-0.2953E-04	-0.1627E-04
.3000E+02	-0.2719E-05	-0.8255E-05	-0.4531E-05
.4000E+02	-0.3705E-05	-0.1131E-05	-0.6195E-05
.5000E+02	-0.1529E-05	-0.4672E-06	-0.2557E-06
.6000E+02	-0.2437E-07	-0.7489E-07	-0.4086E-07
.7000E+02	-0.7115E-08	-0.2186E-07	-0.1192E-07
.8000E+02	-0.1910E-08	-0.5859E-08	-0.3194E-08
.9000E+02	-0.4022E-08	-0.1233E-08	-0.1709E-08
.1000E+03	-0.1350E-09	-0.4076E-09	-0.2235E-09

TABLE 2. (CONTD.)

TOL= 0.10E-02

INITIAL CONDITIONS:

-0.6000E+00 -0.6000E+00 -0.6000E+00

X	Y(1)	Y(2)	Y(3)
.0000E+00	-0.6000E+00	-0.6000E+00	-0.6000E+00
.1000E-02	-0.4233E+00	-0.5997E+00	-0.5996E+00
.2000E-02	0.4439E+00	-0.5991E+00	-0.5985E+00
.3000E-02	0.2930E+00	-0.5983E+00	-0.5971E+00
.4000E-02	0.3739E+00	-0.5974E+00	-0.5956E+00
.5000E-02	0.4159E+00	-0.5964E+00	-0.5939E+00
.6000E-02	0.4371E+00	-0.5953E+00	-0.5922E+00
.7000E-02	0.4455E+00	-0.5943E+00	-0.5905E+00
.8000E-02	0.4455E+00	-0.5933E+00	-0.5887E+00
.9000E-02	0.4434E+00	-0.5922E+00	-0.5870E+00
.1000E-01	0.4382E+00	-0.5912E+00	-0.5853E+00
.2000E-01	0.3779E+00	-0.5814E+00	-0.5690E+00
.3000E-01	0.3248E+00	-0.5721E+00	-0.5537E+00
.4000E-01	0.2813E+00	-0.5634E+00	-0.5394E+00
.5000E-01	0.2457E+00	-0.5552E+00	-0.5261E+00
.6000E-01	0.2148E+00	-0.5473E+00	-0.5135E+00
.7000E-01	0.1880E+00	-0.5399E+00	-0.6015E+00
.8000E-01	0.1647E+00	-0.5327E+00	-0.4902E+00
.9000E-01	0.1441E+00	-0.5259E+00	-0.4795E+00
.1000E+00	0.1259E+00	-0.5193E+00	-0.4692E+00
.1000E+00	0.1259E+00	-0.5193E+00	-0.4692E+00
.2000E+00	0.1552E-01	-0.4640E+00	-0.3872E+00
.3000E+00	-0.8496E-01	-0.4209E+00	-0.3292E+00
.4000E+00	-0.1543E-01	-0.3853E+00	-0.2856E+00
.5000E+00	-0.6913E-01	-0.3548E+00	-0.2515E+00
.6000E+00	-0.7531E-01	-0.3280E+00	-0.2249E+00
.7000E+00	-0.7745E-01	-0.3041E+00	-0.2013E+00
.8000E+00	-0.7745E-01	-0.2826E+00	-0.1823E+00
.9000E+00	-0.7582E-01	-0.2630E+00	-0.1661E+00
.1000E+01	-0.7334E-01	-0.2451E+00	-0.1520E+00
.1000E+01	-0.7334E-01	-0.2451E+00	-0.1520E+00
.2000E+01	-0.4109E-01	-0.1246E+00	-0.7143E-01
.3000E+01	-0.2122E-01	-0.6402E-01	-0.3586E-01
.4000E+01	-0.1090E-01	-0.3296E-01	-0.1829E-01
.5000E+01	-0.5543E-02	-0.1709E-01	-0.9443E-02
.6000E+01	-0.2953E-02	-0.8991E-02	-0.4958E-02
.7000E+01	-0.1582E-02	-0.4802E-02	-0.2644E-02
.8000E+01	-0.8430E-03	-0.2552E-02	-0.1404E-02
.9000E+01	-0.4724E-03	-0.1427E-02	-0.7848E-03
.1000E+02	-0.2534E-03	-0.7683E-03	-0.4222E-03
.1000E+02	-0.2534E-03	-0.7683E-03	-0.4222E-03
.2000E+02	-0.7391E-05	-0.2243E-04	-0.4232E-04
.3000E+02	-0.4186E-05	-0.3604E-05	-0.1977E-05
.4000E+02	-0.7743E-06	-0.2353E-05	-0.1291E-05
.5000E+02	-0.3621E-06	-0.1102E-05	-0.6040E-06
.6000E+02	-0.6550E-07	-0.1708E-06	-0.9320E-07
.7000E+02	-0.3775E-07	-0.1152E-06	-0.6341E-07
.8000E+02	-0.2001E-07	-0.5154E-07	-0.3362E-07
.9000E+02	-0.2538E-08	-0.7913E-08	-0.4287E-08
.1000E+03	-0.8424E-09	-0.1031E-08	-0.5643E-09

TABLE 2. (CONTD.)

TOL= 0.10E-02

INITIAL CONDITIONS:

0.6000E+00 -0.5000E+00 -0.4000E+00

X	Y(1)	Y(2)	Y(3)
.6000E+00	0.5000E+00	-0.5000E+00	-0.4000E+00
.7000E-02	0.4832E+00	-0.4991E+00	-0.3987E+00
.2000E-02	0.4341E-01	-0.4985E+00	-0.3979E+00
.3000E-02	-0.3833E-02	-0.4980E+00	-0.3972E+00
.4000E-02	-0.4915E-01	-0.4975E+00	-0.3966E+00
.5000E-02	-0.2456E-01	-0.4971E+00	-0.3959E+00
.6000E-02	-0.2587E-01	-0.4956E+00	-0.3953E+00
.7000E-02	-0.2793E-01	-0.4951E+00	-0.3947E+00
.8000E-02	-0.2845E-01	-0.4957E+00	-0.3941E+00
.9000E-02	-0.2890E-01	-0.4952E+00	-0.3935E+00
.1000E-01	-0.2939E-01	-0.4947E+00	-0.3929E+00
.2000E-01	-0.3414E-01	-0.4901E+00	-0.3869E+00
.3000E-01	-0.3850E-01	-0.4855E+00	-0.3810E+00
.4000E-01	-0.4273E-01	-0.4812E+00	-0.3753E+00
.5000E-01	-0.4655E-01	-0.4768E+00	-0.3698E+00
.6000E-01	-0.5014E-01	-0.4726E+00	-0.3645E+00
.7000E-01	-0.5345E-01	-0.4684E+00	-0.3592E+00
.8000E-01	-0.5653E-01	-0.4642E+00	-0.3542E+00
.9000E-01	-0.5941E-01	-0.4602E+00	-0.3492E+00
.1000E+00	-0.6205E-01	-0.4552E+00	-0.3444E+00
.1000E+00	-0.6205E-01	-0.4562E+00	-0.3444E+00
.2000E+00	-0.8051E-01	-0.4197E+00	-0.3023E+00
.3000E+00	-0.8913E-01	-0.3878E+00	-0.2688E+00
.4000E+00	-0.9222E-01	-0.3597E+00	-0.2415E+00
.5000E+00	-0.9227E-01	-0.3343E+00	-0.2186E+00
.6000E+00	-0.9036E-01	-0.3112E+00	-0.1990E+00
.7000E+00	-0.8741E-01	-0.2901E+00	-0.1821E+00
.8000E+00	-0.8371E-01	-0.2707E+00	-0.1673E+00
.9000E+00	-0.7959E-01	-0.2528E+00	-0.1542E+00
.1000E+01	-0.7553E-01	-0.2361E+00	-0.1424E+00
.1000E+01	-0.7553E-01	-0.2361E+00	-0.1424E+00
.2000E+01	-0.4007E-01	-0.1210E+00	-0.6911E-01
.3000E+01	-0.2055E-01	-0.6221E-01	-0.3481E-01
.4000E+01	-0.1052E-01	-0.3214E-01	-0.1783E-01
.5000E+01	-0.5490E-02	-0.1663E-01	-0.9184E-02
.6000E+01	-0.2881E-02	-0.8721E-02	-0.4807E-02
.7000E+01	-0.1536E-02	-0.4651E-02	-0.2566E-02
.8000E+01	-0.8357E-03	-0.2535E-02	-0.1395E-02
.9000E+01	-0.4552E-03	-0.1352E-02	-0.7489E-03
.1000E+02	-0.2351E-03	-0.7174E-03	-0.3943E-03
.1000E+02	-0.2351E-03	-0.7174E-03	-0.3943E-03
.2000E+02	-0.6512E-05	-0.2007E-04	-0.1102E-04
.3000E+02	-0.1978E-05	-0.6006E-05	-0.3296E-05
.4000E+02	-0.6821E-06	-0.2074E-05	-0.1138E-05
.5000E+02	-0.1404E-06	-0.4298E-06	-0.2351E-06
.6000E+02	-0.5926E-07	-0.1816E-06	-0.9926E-07
.7000E+02	-0.9791E-08	-0.3022E-07	-0.1645E-07
.8000E+02	-0.4736E-08	-0.1461E-07	-0.7955E-08
.9000E+02	-0.6335E-09	-0.1951E-08	-0.1061E-08
.1000E+03	-0.8422E-10	-0.2539E-09	-0.1392E-09

TABLE 3. A TRAJECTORY CALCULATED FOR TWO-TEMPERATURE MODEL WITHOUT DELAYED NEUTRONS

4.500000 , 0.0000000E+00, 2.000000 , 0.5000000 , -0.500000 ,

TOL= 0.10E-02

INITIAL CONDITIONS:

-0.8000E+00 -0.8000E+00 -0.8000E+00

X	Y(1)	Y(2)	Y(3)
.10000E+00	-0.7015E+00	-0.7935E+00	-0.7986E+00
.20000E+00	-0.6535E+00	-0.7713E+00	-0.7897E+00
.30000E+00	-0.6321E+00	-0.7275E+00	-0.7671E+00
.40000E+00	-0.6541E-02	-0.6531E+00	-0.7238E+00
.50000E+00	0.4533E+00	-0.5356E+00	-0.6505E+00
.60000E+00	0.1104E+01	-0.3595E+00	-0.5354E+00
.70000E+00	0.1909E+01	-0.1097E+00	-0.3643E+00
.80000E+00	0.2773E+01	0.2196E+00	-0.1219E+00
.90000E+00	0.3435E+01	0.5090E+00	0.1845E+00
.10000E+01	0.3526E+01	0.9970E+00	0.5413E+00
.11000E+01	0.2843E+01	0.1289E+01	0.6898E+00
.12000E+01	0.1653E+01	0.1408E+01	0.7155E+00
.13000E+01	0.5453E+00	0.1353E+01	0.7284E+00
.14000E+01	-0.4935E+00	0.1184E+01	0.7273E+00
.15000E+01	-0.8898E+00	0.9575E+00	0.7157E+00
.16000E+01	-0.7825E+00	0.7453E+00	0.6983E+00
.17000E+01	-0.8750E+00	0.5366E+00	0.6786E+00
.18000E+01	-0.9206E+00	0.3478E+00	0.6590E+00
.19000E+01	-0.9442E+00	0.1800E+00	0.4029E+00
.20000E+01	-0.9557E+00	0.3228E-01	0.2333E+00
.21000E+01	-0.9532E+00	-0.9719E-01	0.8146E-01
.22000E+01	-0.9661E+00	-0.2103E+00	-0.5287E-01
.23000E+01	-0.9654E+00	-0.3088E+00	-0.1709E+00
.24000E+01	-0.9645E+00	-0.3943E+00	-0.2740E+00
.25000E+01	-0.9505E+00	-0.4683E+00	-0.3638E+00
.26000E+01	-0.9537E+00	-0.5319E+00	-0.4416E+00
.27000E+01	-0.9435E+00	-0.5851E+00	-0.5086E+00
.28000E+01	-0.9284E+00	-0.6315E+00	-0.5658E+00
.29000E+01	-0.9053E+00	-0.6586E+00	-0.6139E+00
.30000E+01	-0.8742E+00	-0.6973E+00	-0.6534E+00
.31000E+01	-0.8272E+00	-0.7170E+00	-0.6841E+00
.32000E+01	-0.7532E+00	-0.7265E+00	-0.7057E+00
.33000E+01	-0.6570E+00	-0.7235E+00	-0.7169E+00
.34000E+01	-0.5086E+00	-0.7044E+00	-0.7154E+00
.35000E+01	-0.2929E+00	-0.6537E+00	-0.6978E+00
.36000E+01	0.1571E-01	-0.5935E+00	-0.6587E+00
.37000E+01	0.4433E+00	-0.4836E+00	-0.5906E+00
.38000E+01	0.4010E+01	-0.3218E+00	-0.4839E+00
.39000E+01	0.1694E+01	-0.9756E-01	-0.3277E+00
.40000E+01	0.2402E+01	0.1908E+00	-0.1138E+00
.41000E+01	0.2927E+01	0.5242E+00	0.1567E+00
.42000E+01	0.2999E+01	0.8529E+00	0.4633E+00
.43000E+01	0.2475E+01	0.1104E+01	0.7613E+00
.44000E+01	0.1544E+01	0.1218E+01	0.9923E+00
.45000E+01	0.6000E+00	0.1187E+01	0.1113E+01
.46000E+01	-0.8521E-01	0.1052E+01	0.1115E+01
.47000E+01	-0.4930E+00	0.8674E+00	0.1024E+01
.48000E+01	-0.7104E+00	0.6594E+00	0.8761E+00
.49000E+01	-0.8231E+00	0.4784E+00	0.7022E+00
.50000E+01	-0.8822E+00	0.3028E+00	0.5234E+00
.51000E+01	-0.9140E+00	0.1452E+00	0.3520E+00
.52000E+01	-0.9313E+00	0.6472E-02	0.1938E+00
.53000E+01	-0.9404E+00	-0.1175E+00	0.5104E-01
.54000E+01	-0.9441E+00	-0.2250E+00	-0.7594E-01
.55000E+01	-0.9439E+00	-0.3187E+00	-0.1878E+00
.56000E+01	-0.9402E+00	-0.3999E+00	-0.2858E+00
.57000E+01	-0.9323E+00	-0.4698E+00	-0.3709E+00
.58000E+01	-0.9209E+00	-0.5292E+00	-0.4443E+00
.59000E+01	-0.9031E+00	-0.5789E+00	-0.5070E+00
.60000E+01	-0.8771E+00	-0.6193E+00	-0.5596E+00
.61000E+01	-0.8395E+00	-0.6502E+00	-0.6026E+00
.62000E+01	-0.7852E+00	-0.6710E+00	-0.6358E+00
.63000E+01	-0.7058E+00	-0.6803E+00	-0.6586E+00
.64000E+01	-0.5937E+00	-0.6758E+00	-0.6699E+00
.65000E+01	-0.4312E+00	-0.6536E+00	-0.6671E+00
.66000E+01	-0.2002E+00	-0.6082E+00	-0.6468E+00
.67000E+01	0.4214E+00	-0.5322E+00	-0.6034E+00
.68000E+01	0.1537E+00	-0.4159E+00	-0.5300E+00

TABLE 3. (CONTD.)

.6900E+01	0.1121E+01	-0.2496E+00	-0.4178E+00
.7000E+01	0.1728E+01	-0.2645E-01	-0.2582E+00
.7100E+01	0.2323E+01	0.2494E+00	-0.4667E-01
.7200E+01	0.2892E+01	0.5536E+00	0.2108E+00
.7300E+01	0.3426E+01	0.8354E+00	0.4902E+00
.7400E+01	0.3907E+01	0.1036E+01	0.7481E+00
.7500E+01	0.4331E+01	0.1111E+01	0.9360E+00
.7600E+01	0.4693E+01	0.1063E+01	0.1022E+01
.7700E+01	0.4992E+01	0.9318E+00	0.1007E+01
.7800E+01	0.5225E+01	0.7592E+00	0.9127E+00
.7900E+01	0.5388E+01	0.5763E+00	0.7711E+00
.8000E+01	0.5483E+01	0.3998E+00	0.6083E+00
.8100E+01	0.5513E+01	0.2370E+00	0.4420E+00
.8200E+01	0.5480E+01	0.9040E-01	0.2827E+00
.8300E+01	0.5387E+01	-0.3986E-01	0.1355E+00
.8400E+01	0.5240E+01	-0.1545E+00	0.2491E-02
.8500E+01	0.5044E+01	-0.2550E+00	-0.1159E+00
.8600E+01	0.4805E+01	-0.3422E+00	-0.2203E+00
.8700E+01	0.4530E+01	-0.4175E+00	-0.3114E+00
.8800E+01	0.4225E+01	-0.4817E+00	-0.3903E+00
.8900E+01	0.3897E+01	-0.5356E+00	-0.4579E+00
.9000E+01	0.3550E+01	-0.5795E+00	-0.5148E+00
.9100E+01	0.3223E+01	-0.6134E+00	-0.5615E+00
.9200E+01	0.2922E+01	-0.6367E+00	-0.5978E+00
.9300E+01	0.2685E+01	-0.6481E+00	-0.6234E+00
.9400E+01	0.2575E+01	-0.6454E+00	-0.6369E+00
.9500E+01	0.2655E+01	-0.6250E+00	-0.6361E+00
.9600E+01	0.2929E+01	-0.5820E+00	-0.6178E+00
.9700E+01	0.3444E+01	-0.5093E+00	-0.5770E+00
.9800E+01	0.4222E+01	-0.3986E+00	-0.5071E+00
.9900E+01	0.5031E+01	-0.2414E+00	-0.4007E+00
.1000E+02	0.5659E+01	-0.3218E-01	-0.2501E+00
.1010E+02	0.6211E+01	0.2244E+00	-0.5194E-01
.1020E+02	0.6487E+01	0.5062E+00	0.1878E+00
.1030E+02	0.6443E+01	0.7691E+00	0.4472E+00
.1040E+02	0.6054E+01	0.9587E+00	0.6880E+00
.1050E+02	0.5421E+01	0.1036E+01	0.8567E+00
.1060E+02	0.4690E+01	0.1001E+01	0.9538E+00
.1070E+02	0.3955E+01	0.8851E+00	0.9465E+00
.1080E+02	0.3511E+01	0.7250E+00	0.8643E+00
.1090E+02	0.3583E+01	0.6534E+00	0.7345E+00
.1100E+02	0.3745E+01	0.3844E+00	0.5817E+00
.1110E+02	0.3395E+01	0.2271E+00	0.4234E+00
.1120E+02	0.3752E+01	0.8457E-01	0.2702E+00
.1130E+02	0.3895E+01	-0.4258E-01	0.1277E+00
.1140E+02	0.3957E+01	-0.1548E+00	-0.4733E-02
.1150E+02	0.3999E+01	-0.2532E+00	-0.1173E+00
.1160E+02	0.4074E+01	-0.3386E+00	-0.2193E+00
.1170E+02	0.3994E+01	-0.4122E+00	-0.3085E+00
.1180E+02	0.3854E+01	-0.4747E+00	-0.3855E+00
.1190E+02	0.3638E+01	-0.5256E+00	-0.4513E+00
.1200E+02	0.3321E+01	-0.5683E+00	-0.5062E+00
.1210E+02	0.7857E+00	-0.5994E+00	-0.5506E+00
.1220E+02	0.7218E+00	-0.6192E+00	-0.5842E+00
.1230E+02	0.6299E+00	-0.6260E+00	-0.6062E+00
.1240E+02	0.4999E+00	-0.6173E+00	-0.6152E+00
.1250E+02	0.3179E+00	-0.5890E+00	-0.6085E+00
.1260E+02	0.6694E-01	-0.5358E+00	-0.5825E+00
.1270E+02	0.2599E+00	-0.4505E+00	-0.5318E+00
.1280E+02	0.7021E+00	-0.3252E+00	-0.4500E+00
.1290E+02	0.1217E+01	-0.1531E+00	-0.3298E+00
.1300E+02	0.1757E+01	0.6706E-01	-0.1659E+00
.1310E+02	0.2201E+01	0.3242E+00	0.4134E-01
.1320E+02	0.2382E+01	0.5887E+00	0.2802E+00
.1330E+02	0.2170E+01	0.8140E+00	0.5233E+00
.1340E+02	0.1598E+01	0.9533E+00	0.7315E+00
.1350E+02	0.8743E+00	0.9838E+00	0.8680E+00
.1360E+02	0.2275E+00	0.9179E+00	0.9149E+00
.1370E+02	-0.2343E+00	0.7887E+00	0.8783E+00
.1380E+02	-0.6223E+00	0.5297E+00	0.7801E+00
.1390E+02	-0.9902E+00	0.4640E+00	0.6456E+00
.1400E+02	-0.7850E+00	0.3044E+00	0.4954E+00
.1410E+02	-0.8407E+00	0.1571E+00	0.3437E+00
.1420E+02	-0.8717E+00	0.2409E-01	0.1988E+00
.1430E+02	-0.8886E+00	-0.9427E-01	0.6505E-01
.1440E+02	-0.8995E+00	-0.1985E+00	-0.5579E-01
.1450E+02	-0.8956E+00	-0.2896E+00	-0.1633E+00
.1460E+02	-0.8910E+00	-0.3683E+00	-0.2579E+00
.1470E+02	-0.8791E+00	-0.4356E+00	-0.3402E+00
.1480E+02	-0.8597E+00	-0.4920E+00	-0.4108E+00
.1490E+02	-0.8339E+00	-0.5378E+00	-0.4703E+00
.1500E+02	-0.7892E+00	-0.5730E+00	-0.5189E+00
.1510E+02	-0.7298E+00	-0.5970E+00	-0.5567E+00
.1520E+02	-0.6545E+00	-0.6083E+00	-0.5830E+00
.1530E+02	-0.5277E+00	-0.6048E+00	-0.5965E+00
.1540E+02	-0.3623E+00	-0.5839E+00	-0.5952E+00
.1550E+02	-0.1359E+00	-0.5381E+00	-0.5757E+00

TABLE 3.(CONTD.)

.7750E+02	-0.7885E+00	-0.4550E+00	-0.8796E+00
.7760E+02	-0.7485E+00	-0.4956E+00	-0.4343E+00
.7770E+02	-0.6916E+00	-0.5245E+00	-0.4776E+00
.7780E+02	-0.6121E+00	-0.5406E+00	-0.5091E+00
.7790E+02	-0.5023E+00	-0.5421E+00	-0.6276E+00
.7800E+02	-0.3624E+00	-0.5261E+00	-0.6315E+00
.7810E+02	-0.1510E+00	-0.4890E+00	-0.6180E+00
.7820E+02	0.1429E+00	-0.4259E+00	-0.4837E+00
.7830E+02	0.4458E+00	-0.3314E+00	-0.4239E+00
.7840E+02	0.8406E+00	-0.2008E+00	-0.3340E+00
.7850E+02	0.1254E+01	-0.3269E-01	-0.2099E+00
.7860E+02	0.1647E+01	0.1671E+00	-0.5119E-01
.7870E+02	0.1881E+01	0.3814E+00	0.1357E+00
.7880E+02	0.1855E+01	0.5809E+00	0.3347E+00
.7890E+02	0.1555E+01	0.7307E+00	0.5200E+00
.7900E+02	0.1053E+01	0.8043E+00	0.6627E+00
.7910E+02	0.5137E+00	0.7962E+00	0.7417E+00
.7920E+02	0.4479E-01	0.7215E+00	0.7509E+00
.7930E+02	-0.2952E+00	0.5045E+00	0.6996E+00
.7940E+02	-0.5184E+00	0.4675E+00	0.6050E+00
.7950E+02	-0.6574E+00	0.3255E+00	0.4853E+00
.7960E+02	-0.7420E+00	0.1905E+00	0.3551E+00
.7970E+02	-0.7926E+00	0.5424E-01	0.2246E+00
.7980E+02	-0.8215E+00	-0.5016E-01	0.1001E+00
.7990E+02	-0.8353E+00	-0.1521E+00	-0.4500E-01
.8000E+02	-0.8402E+00	-0.2418E+00	-0.1490E+00
.8010E+02	-0.8352E+00	-0.3195E+00	-0.2114E+00
.8020E+02	-0.8213E+00	-0.3858E+00	-0.2921E+00
.8030E+02	-0.7975E+00	-0.4459E+00	-0.3614E+00
.8040E+02	-0.7515E+00	-0.4848E+00	-0.4143E+00
.8050E+02	-0.7102E+00	-0.5172E+00	-0.4660E+00
.8060E+02	-0.6379E+00	-0.5372E+00	-0.5015E+00
.8070E+02	-0.5373E+00	-0.5431E+00	-0.6235E+00
.8080E+02	-0.4007E+00	-0.5326E+00	-0.6318E+00
.8090E+02	-0.2155E+00	-0.5021E+00	-0.6238E+00
.8100E+02	0.2905E-01	-0.4471E+00	-0.4959E+00
.8110E+02	0.3417E+00	-0.3623E+00	-0.4441E+00
.8120E+02	0.7203E+00	-0.2426E+00	-0.3635E+00
.8130E+02	0.1141E+01	-0.8535E-01	-0.2496E+00
.8140E+02	0.1545E+01	0.1063E+00	-0.1008E+00
.8150E+02	0.1834E+01	0.3188E+00	0.7906E-01
.8160E+02	0.1899E+01	0.5262E+00	0.2767E+00
.8170E+02	0.1673E+01	0.6937E+00	0.4689E+00
.8180E+02	0.1221E+01	0.7911E+00	0.6267E+00
.8190E+02	0.6597E+00	0.8051E+00	0.7256E+00
.8200E+02	0.1639E+00	0.7485E+00	0.7548E+00
.8210E+02	-0.2092E+00	0.6413E+00	0.7196E+00
.8220E+02	-0.4533E+00	0.5083E+00	0.6357E+00
.8230E+02	-0.6235E+00	0.3673E+00	0.5217E+00
.8240E+02	-0.7211E+00	0.2292E+00	0.3933E+00
.8250E+02	-0.7803E+00	0.9982E-01	0.2621E+00
.8260E+02	-0.8147E+00	-0.1811E-01	0.1354E+00
.8270E+02	-0.8331E+00	-0.1237E+00	0.1741E-01
.8280E+02	-0.8399E+00	-0.2169E+00	-0.8989E-01
.8290E+02	-0.8371E+00	-0.2980E+00	-0.1857E+00
.8300E+02	-0.8251E+00	-0.3675E+00	-0.2597E+00
.8310E+02	-0.8054E+00	-0.4259E+00	-0.3423E+00
.8320E+02	-0.7733E+00	-0.4731E+00	-0.4036E+00
.8330E+02	-0.7258E+00	-0.5089E+00	-0.4535E+00
.8340E+02	-0.6612E+00	-0.5326E+00	-0.4920E+00
.8350E+02	-0.5701E+00	-0.5428E+00	-0.5182E+00
.8360E+02	-0.4448E+00	-0.5374E+00	-0.5308E+00
.8370E+02	-0.2748E+00	-0.5131E+00	-0.5278E+00
.8380E+02	-0.4852E-01	-0.4658E+00	-0.5061E+00
.8390E+02	0.2439E+00	-0.3902E+00	-0.4618E+00
.8400E+02	0.5045E+00	-0.2811E+00	-0.3900E+00
.8410E+02	0.1017E+01	-0.1347E+00	-0.2861E+00
.8420E+02	0.1434E+01	0.4780E-01	-0.1474E+00
.8430E+02	0.1756E+01	0.2553E+00	0.2434E-01
.8440E+02	0.1998E+01	0.4684E+00	0.2188E+00
.8450E+02	0.1777E+01	0.6511E+00	0.4153E+00
.8460E+02	0.1373E+01	0.7710E+00	0.5860E+00
.8470E+02	0.8224E+00	0.8098E+00	0.7037E+00
.8480E+02	0.3030E+00	0.7712E+00	0.7532E+00
.8490E+02	-0.1132E+00	0.6758E+00	0.7354E+00
.8500E+02	-0.4077E+00	0.5482E+00	0.6637E+00
.8510E+02	-0.6845E+00	0.4081E+00	0.5567E+00
.8520E+02	-0.8978E+00	0.2684E+00	0.4311E+00
.8530E+02	-0.7552E+00	0.1362E+00	0.2999E+00
.8540E+02	-0.8055E+00	0.1478E-01	0.1713E+00
.8550E+02	-0.8290E+00	-0.9439E-01	0.6054E-01
.8560E+02	-0.8388E+00	-0.1911E+00	-0.6998E-01
.8570E+02	-0.8339E+00	-0.2757E+00	-0.1591E+00
.8580E+02	-0.8302E+00	-0.3486E+00	-0.2466E+00
.8590E+02	-0.8123E+00	-0.4101E+00	-0.3224E+00
.8600E+02	-0.7838E+00	-0.4605E+00	-0.3869E+00
.8610E+02	-0.7419E+00	-0.4996E+00	-0.4402E+00
.8620E+02	-0.6823E+00	-0.5269E+00	-0.4820E+00
.8630E+02	-0.5993E+00	-0.5413E+00	-0.5118E+00

TABLE 3. (CONTD.)

.8640E+02	-0.4849E+00	-0.5406E+00	-0.6286E+00
.8650E+02	-0.3290E+00	-0.5222E+00	-0.6304E+00
.8660E+02	-0.1201E+00	-0.4820E+00	-0.6144E+00
.8670E+02	0.1525E+00	-0.4151E+00	-0.4771E+00
.8680E+02	0.4940E+00	-0.3151E+00	-0.4137E+00
.8690E+02	0.8947E+00	-0.1807E+00	-0.3195E+00
.8700E+02	0.1317E+01	-0.8135E-02	-0.1909E+00
.8710E+02	0.1686E+01	0.1946E+00	-0.2811E-01
.8720E+02	0.1893E+01	0.4086E+00	0.1613E+00
.8730E+02	0.1839E+01	0.6034E+00	0.3599E+00
.8740E+02	0.1505E+01	0.7442E+00	0.5410E+00
.8750E+02	0.9899E+00	0.8069E+00	0.6762E+00
.8760E+02	0.4450E+00	0.7890E+00	0.7460E+00
.8770E+02	-0.6701E-02	0.7075E+00	0.7466E+00
.8780E+02	-0.3298E+00	0.5869E+00	0.6887E+00
.8790E+02	-0.5401E+00	0.4488E+00	0.5900E+00
.8800E+02	-0.6705E+00	0.3081E+00	0.4682E+00
.8810E+02	-0.7498E+00	0.1732E+00	0.3376E+00
.8820E+02	-0.7970E+00	0.4853E-01	0.2077E+00
.8830E+02	-0.8239E+00	-0.6421E-01	0.8436E-01
.8840E+02	-0.8370E+00	-0.1645E+00	-0.2928E-01
.8850E+02	-0.8397E+00	-0.2525E+00	-0.1317E+00
.8860E+02	-0.8335E+00	-0.3287E+00	-0.2225E+00
.8870E+02	-0.8184E+00	-0.3935E+00	-0.3017E+00
-0.4471E+00	-0.3595E+00		
.8890E+02	-0.7554E+00	-0.4895E+00	-0.4259E+00
.8900E+02	-0.7015E+00	-0.5203E+00	-0.4711E+00
.8910E+02	-0.6259E+00	-0.5385E+00	-0.5044E+00
.8920E+02	-0.5214E+00	-0.5424E+00	-0.5251E+00
.8930E+02	-0.3785E+00	-0.5294E+00	-0.5314E+00
.8940E+02	-0.1851E+00	-0.4959E+00	-0.5209E+00
.8950E+02	0.6725E-01	-0.4373E+00	-0.4901E+00
.8960E+02	0.3890E+00	-0.3481E+00	-0.4347E+00
.8970E+02	0.7750E+00	-0.2234E+00	-0.3499E+00
.8980E+02	0.1197E+01	-0.5124E-01	-0.2314E+00
.8990E+02	0.1591E+01	0.1341E+00	-0.7805E-01
.9000E+02	0.1855E+01	0.3476E+00	0.1050E+00

NOTE: INTERMEDIATE STEPS HAVE BEEN OMITTED.

TABLE 4. A TRAJECTORY CALCULATED FOR TWO-TEMPERATURE MODEL WITHOUT DELAYED NEUTRONS

4.500000 , 0.0000000E+00, 2.000000 , 0.5000000 , -0.500

TOL= 0.10E-02

INITIAL CONDITIONS:

0.2000E+00 0.1000E+00 0.1000E+00

X	Y(1)	Y(2)	Y(3)
.1000E+00	0.1429E+00	0.1096E+00	0.1035E+00
.2000E+00	0.8650E-01	0.1100E+00	0.1073E+00
.3000E+00	0.2933E-01	0.1027E+00	0.1069E+00
.4000E+00	-0.2640E-01	0.8883E-01	0.1005E+00
.5000E+00	-0.7785E-01	0.6975E-01	0.8833E-01
.6000E+00	-0.1223E+00	0.4691E-01	0.7112E-01
.7000E+00	-0.1559E+00	0.2183E-01	0.5004E-01
.8000E+00	-0.1835E+00	-0.4009E-02	0.2645E-01
.9000E+00	-0.2027E+00	-0.2922E-01	0.1762E-02
.1000E+01	-0.2094E+00	-0.5256E-01	-0.2268E-01
.1100E+01	-0.2055E+00	-0.7295E-01	-0.4565E-01
.1200E+01	-0.1923E+00	-0.8947E-01	-0.6602E-01
.1300E+01	-0.1699E+00	-0.1014E+00	-0.8287E-01
.1400E+01	-0.1390E+00	-0.1080E+00	-0.9538E-01
.1500E+01	-0.9782E-01	-0.1090E+00	-0.1029E+00
.1600E+01	-0.5056E-01	-0.1049E+00	-0.1050E+00
.1700E+01	-0.4972E-02	-0.9308E-01	-0.1013E+00
.1800E+01	0.5745E-01	-0.7645E-01	-0.9184E-01
.1900E+01	0.1126E+00	-0.5472E-01	-0.7681E-01
.2000E+01	0.1635E+00	-0.2887E-01	-0.5675E-01
.2100E+01	0.2053E+00	-0.2959E-03	-0.3258E-01
.2200E+01	0.2356E+00	0.2925E-01	-0.5591E-02
.2300E+01	0.2511E+00	0.5773E-01	0.2257E-01
.2400E+01	0.2475E+00	0.8304E-01	0.5001E-01
.2500E+01	0.2251E+00	0.1032E+00	0.7476E-01
.2600E+01	0.1883E+00	0.1156E+00	0.9493E-01
.2700E+01	0.1387E+00	0.1223E+00	0.1090E+00
.2800E+01	0.7952E-01	0.1200E+00	0.1160E+00
.2900E+01	0.1820E-01	0.1100E+00	0.1154E+00
.3000E+01	-0.4152E-01	0.9338E-01	0.1075E+00
.3100E+01	-0.9550E-01	0.7146E-01	0.9310E-01
.3200E+01	-0.1426E+00	0.4585E-01	0.7333E-01
.3300E+01	-0.1797E+00	0.1822E-01	0.4964E-01
.3400E+01	-0.2063E+00	-0.9859E-02	0.2357E-01
.3500E+01	-0.2222E+00	-0.3692E-01	-0.3349E-02
.3600E+01	-0.2272E+00	-0.6169E-01	-0.2969E-01
.3700E+01	-0.2215E+00	-0.8305E-01	-0.5414E-01
.3800E+01	-0.2054E+00	-0.1001E+00	-0.7558E-01
.3900E+01	-0.1792E+00	-0.1120E+00	-0.9303E-01
.4000E+01	-0.1433E+00	-0.1182E+00	-0.1057E+00
.4100E+01	-0.9853E-01	-0.1182E+00	-0.1128E+00
.4200E+01	-0.4643E-01	-0.1117E+00	-0.1141E+00
.4300E+01	0.1137E-01	-0.9884E-01	-0.1090E+00
.4400E+01	0.7209E-01	-0.7983E-01	-0.9777E-01
.4500E+01	0.1322E+00	-0.5536E-01	-0.8049E-01
.4600E+01	0.1873E+00	-0.2654E-01	-0.5783E-01
.4700E+01	0.2327E+00	0.5057E-02	-0.3083E-01
.4800E+01	0.2637E+00	0.3743E-01	-0.9648E-03
.4900E+01	0.2765E+00	0.6829E-01	0.2992E-01
.5000E+01	0.2692E+00	0.9529E-01	0.5969E-01
.5100E+01	0.2419E+00	0.1163E+00	0.8615E-01
.5200E+01	0.1971E+00	0.1295E+00	0.1072E+00
.5300E+01	0.1392E+00	0.1341E+00	0.1213E+00
.5400E+01	0.7385E-01	0.1299E+00	0.1274E+00
.5500E+01	0.6515E-02	0.1174E+00	0.1252E+00
.5600E+01	-0.5785E-01	0.9787E-01	0.1151E+00
.5700E+01	-0.1155E+00	0.7296E-01	0.9806E-01
.5800E+01	-0.1639E+00	0.4445E-01	0.7547E-01
.5900E+01	-0.2016E+00	0.1415E-01	0.4899E-01
.6000E+01	-0.2279E+00	-0.1625E-01	0.2029E-01
.6100E+01	-0.2427E+00	-0.4523E-01	-0.6959E-02
.6200E+01	-0.2459E+00	-0.7145E-01	-0.3726E-01
.6300E+01	-0.2379E+00	-0.9380E-01	-0.6325E-01
.6400E+01	-0.2188E+00	-0.1113E+00	-0.8578E-01
.6500E+01	-0.1889E+00	-0.1232E+00	-0.1038E+00
.6600E+01	-0.1489E+00	-0.1289E+00	-0.1166E+00
.6700E+01	-0.9952E-01	-0.1279E+00	-0.1234E+00
.6800E+01	-0.4205E-01	-0.1199E+00	-0.1237E+00
.6900E+01	0.2140E-01	-0.1050E+00	-0.1173E+00
.7000E+01	0.8785E-01	-0.8340E-01	-0.1041E+00
.7100E+01	0.1533E+00	-0.5598E-01	-0.8438E-01

TABLE 4. (CONTD.)

.7200E+01	0.2129E+00	-0.2396E-01	-0.5893E-01
.7300E+01	0.2511E+00	0.1089E-01	-0.2889E-01
.7400E+01	0.2929E+00	0.4632E-01	0.4991E-02
.7500E+01	0.3039E+00	0.7974E-01	0.3791E-01
.7600E+01	0.2922E+00	0.1085E+00	0.7019E-01
.7700E+01	0.2585E+00	0.1303E+00	0.9846E-01
.7800E+01	0.2059E+00	0.1433E+00	0.1205E+00
.7900E+01	0.1400E+00	0.1467E+00	0.1345E+00
.8000E+01	0.6714E-01	0.1403E+00	0.1397E+00
.8100E+01	-0.5453E-02	0.1251E+00	0.1356E+00
.8200E+01	-0.7554E-01	0.1025E+00	0.1231E+00
.8300E+01	-0.1363E+00	0.7437E-01	0.1032E+00
.8400E+01	-0.1854E+00	0.4280E-01	0.7758E-01
.8500E+01	-0.2247E+00	0.9710E-02	0.4814E-01
.8600E+01	-0.2505E+00	-0.2310E-01	0.1670E-01
.8700E+01	-0.2641E+00	-0.5405E-01	-0.1498E-01
.8800E+01	-0.2655E+00	-0.8179E-01	-0.4531E-01
.8900E+01	-0.2550E+00	-0.1052E+00	-0.7290E-01
.9000E+01	-0.2329E+00	-0.1232E+00	-0.9655E-01
.9100E+01	-0.1994E+00	-0.1352E+00	-0.1152E+00
.9200E+01	-0.1551E+00	-0.1403E+00	-0.1281E+00
.9300E+01	-0.1008E+00	-0.1382E+00	-0.1345E+00
.9400E+01	-0.3774E-01	-0.1287E+00	-0.1339E+00
.9500E+01	0.3176E-01	-0.1116E+00	-0.1260E+00
.9600E+01	0.1044E+00	-0.8731E-01	-0.1108E+00
.9700E+01	0.1758E+00	-0.5675E-01	-0.8860E-01
.9800E+01	0.2403E+00	-0.2129E-01	-0.6019E-01
.9900E+01	0.2918E+00	0.1706E-01	-0.2689E-01
.1000E+02	0.3244E+00	0.5582E-01	0.9449E-02

.1010E+02	0.3334E+00	0.9202E-01	0.4645E-01
.1020E+02	0.3159E+00	0.1227E+00	0.8143E-01
.1030E+02	0.2761E+00	0.1454E+00	0.1117E+00
.1040E+02	0.2151E+00	0.1581E+00	0.1347E+00
.1050E+02	0.1405E+00	0.1600E+00	0.1487E+00
.1060E+02	0.5956E-01	0.1513E+00	0.1527E+00
.1070E+02	-0.2046E-01	0.1332E+00	0.1467E+00
.1080E+02	-0.9436E-01	0.1073E+00	0.1315E+00
.1090E+02	-0.1583E+00	0.7583E-01	0.1085E+00
.1100E+02	-0.2130E+00	0.4105E-01	0.7979E-01
.1110E+02	-0.2487E+00	0.5055E-02	0.4725E-01
.1120E+02	-0.2741E+00	-0.3026E-01	0.1294E-01
.1130E+02	-0.2854E+00	-0.5327E-01	-0.2128E-01
.1140E+02	-0.2865E+00	-0.9258E-01	-0.5373E-01
.1150E+02	-0.2731E+00	-0.1170E+00	-0.8300E-01
.1160E+02	-0.2480E+00	-0.1357E+00	-0.1078E+00
.1170E+02	-0.2109E+00	-0.1477E+00	-0.1272E+00
.1180E+02	-0.1623E+00	-0.1524E+00	-0.1403E+00
.1190E+02	-0.1028E+00	-0.1493E+00	-0.1464E+00
.1200E+02	-0.8395E-01	-0.1381E+00	-0.1449E+00
.1210E+02	0.4203E-01	-0.1188E+00	-0.1355E+00
.1220E+02	0.1215E+00	-0.9175E-01	-0.1182E+00
.1230E+02	0.1994E+00	-0.5788E-01	-0.9336E-01
.1240E+02	0.2695E+00	-0.1875E-01	-0.6182E-01
.1250E+02	0.3247E+00	0.2342E-01	-0.2503E-01
.1260E+02	0.3684E+00	0.5576E-01	0.1491E-01
.1270E+02	0.3654E+00	0.1050E+00	0.5536E-01
.1280E+02	0.3437E+00	0.1378E+00	0.9329E-01
.1290E+02	0.2951E+00	0.1615E+00	0.1257E+00
.1300E+02	0.2250E+00	0.1739E+00	0.1498E+00
.1310E+02	0.1410E+00	0.1743E+00	0.1637E+00
.1320E+02	0.5179E-01	0.1636E+00	0.1666E+00
.1330E+02	-0.3515E-01	0.1419E+00	0.1585E+00
.1340E+02	-0.1140E+00	0.1125E+00	0.1405E+00
.1350E+02	-0.1810E+00	0.7754E-01	0.1144E+00
.1360E+02	-0.2344E+00	0.3941E-01	0.8230E-01
.1370E+02	-0.2735E+00	0.3895E-03	0.4651E-01
.1380E+02	-0.2985E+00	-0.3755E-01	0.9201E-02
.1390E+02	-0.3097E+00	-0.7271E-01	-0.2766E-01
.1400E+02	-0.3075E+00	-0.1037E+00	-0.6235E-01
.1410E+02	-0.2924E+00	-0.1294E+00	-0.9338E-01
.1420E+02	-0.2644E+00	-0.1487E+00	-0.1195E+00
.1430E+02	-0.2238E+00	-0.1609E+00	-0.1397E+00
.1440E+02	-0.1709E+00	-0.1653E+00	-0.1531E+00
.1450E+02	-0.1062E+00	-0.1612E+00	-0.1589E+00
.1460E+02	-0.3125E-01	-0.1485E+00	-0.1566E+00
.1470E+02	0.5158E-01	-0.1269E+00	-0.1458E+00
.1480E+02	0.1384E+00	-0.9791E-01	-0.1264E+00
.1490E+02	0.2235E+00	-0.5969E-01	-0.9891E-01
.1500E+02	0.3000E+00	-0.1656E-01	-0.6411E-01
.1510E+02	0.3597E+00	0.2950E-01	-0.2364E-01
.1520E+02	0.3949E+00	0.7587E-01	0.2018E-01
.1530E+02	0.4001E+00	0.1135E+00	0.6438E-01
.1540E+02	0.3730E+00	0.1538E+00	0.1056E+00
.1550E+02	0.3152E+00	0.1785E+00	0.1404E+00
.1560E+02	0.2362E+00	0.1908E+00	0.1658E+00
.1570E+02	0.1423E+00	0.1897E+00	0.1798E+00
.1580E+02	0.4410E-01	0.1760E+00	0.1815E+00
.1590E+02	-0.4999E-01	0.1514E+00	0.1712E+00

TABLE 4. (CONTD.)

.1041E+03	-0.4639E+00	-0.5322E+00	-0.5220E+00
.1042E+03	-0.8036E+00	-0.5118E+00	-0.5221E+00
.1043E+03	-0.9007E-01	-0.4693E+00	-0.6043E+00
.1044E+03	0.1855E+00	-0.3998E+00	-0.4647E+00
.1045E+03	0.5300E+00	-0.2982E+00	-0.3989E+00
.1046E+03	0.9275E+00	-0.1605E+00	-0.3022E+00
.1047E+03	0.1339E+01	0.4298E-01	-0.1716E+00
.1048E+03	0.1685E+01	0.2142E+00	-0.8160E-02
.1049E+03	0.1851E+01	0.4234E+00	0.1708E+00
.1050E+03	0.1778E+01	0.6097E+00	0.3738E+00
.1051E+03	0.1430E+01	0.7402E+00	0.5476E+00
.1052E+03	0.9174E+00	0.7934E+00	0.6740E+00
.1053E+03	0.3914E+00	0.7690E+00	0.7354E+00
.1054E+03	-0.4092E-01	0.6845E+00	0.7297E+00
.1055E+03	-0.3481E+00	0.5537E+00	0.6683E+00
.1056E+03	-0.5481E+00	0.4272E+00	0.5685E+00
.1057E+03	-0.6728E+00	0.2886E+00	0.4474E+00
.1058E+03	-0.7487E+00	0.1565E+00	0.3184E+00
.1059E+03	-0.7940E+00	0.3428E-01	0.1906E+00
.1060E+03	-0.8197E+00	-0.7610E-01	0.6958E-01
.1061E+03	-0.8317E+00	-0.1742E+00	-0.4177E-01
.1062E+03	-0.8334E+00	-0.2602E+00	-0.1420E+00
.1063E+03	-0.8259E+00	-0.3345E+00	-0.2308E+00
.1064E+03	-0.8092E+00	-0.3974E+00	-0.3080E+00
.1065E+03	-0.7819E+00	-0.4491E+00	-0.3738E+00
.1066E+03	-0.7413E+00	-0.4896E+00	-0.4284E+00
.1067E+03	-0.6837E+00	-0.5183E+00	-0.4716E+00
.1068E+03	-0.6033E+00	-0.5341E+00	-0.5028E+00
.1069E+03	-0.4925E+00	-0.5352E+00	-0.5211E+00
.1070E+03	-0.3417E+00	-0.5188E+00	-0.5246E+00
.1071E+03	-0.1399E+00	-0.4812E+00	-0.5107E+00
.1072E+03	0.1235E+00	-0.4176E+00	-0.4760E+00
.1073E+03	0.4542E+00	-0.3229E+00	-0.4159E+00
.1074E+03	0.8439E+00	-0.1927E+00	-0.3258E+00
.1075E+03	0.1259E+01	-0.2584E-01	-0.2021E+00
.1076E+03	0.1629E+01	0.1714E+00	-0.4463E-01
.1077E+03	0.1852E+01	0.3818E+00	0.1399E+00
.1078E+03	0.1825E+01	0.5754E+00	0.3353E+00
.1079E+03	0.1525E+01	0.7213E+00	0.5162E+00
.1080E+03	0.1032E+01	0.7914E+00	0.6547E+00
.1081E+03	0.4354E+00	0.7819E+00	0.7304E+00
.1082E+03	-0.3750E-01	0.7076E+00	0.7380E+00
.1083E+03	-0.2955E+00	0.5920E+00	0.6866E+00
.1084E+03	-0.5154E+00	0.4570E+00	0.6929E+00
.1085E+03	-0.6530E+00	0.3179E+00	0.4746E+00
.1086E+03	-0.7371E+00	0.1836E+00	0.3461E+00
.1087E+03	-0.7875E+00	0.5891E-01	0.2173E+00
.1088E+03	-0.8155E+00	-0.5414E-01	0.9427E-01
.1089E+03	-0.8311E+00	-0.1549E+00	-0.1942E-01
.1090E+03	-0.8347E+00	-0.2435E+00	-0.1222E+00
.1091E+03	-0.8292E+00	-0.3203E+00	-0.2135E+00
.1092E+03	-0.8145E+00	-0.3856E+00	-0.2932E+00
.1093E+03	-0.7897E+00	-0.4398E+00	-0.3615E+00
.1094E+03	-0.7523E+00	-0.4827E+00	-0.4185E+00
.1095E+03	-0.6999E+00	-0.5140E+00	-0.4642E+00
.1096E+03	-0.6241E+00	-0.5328E+00	-0.4981E+00
.1097E+03	-0.5207E+00	-0.5373E+00	-0.5193E+00
.1098E+03	-0.3795E+00	-0.5259E+00	-0.5262E+00
.1099E+03	-0.1897E+00	-0.4924E+00	-0.5164E+00
.1100E+03	0.9985E-01	-0.4349E+00	-0.4865E+00
.1101E+03	0.3763E+00	-0.3473E+00	-0.4323E+00
.1102E+03	0.7557E+00	-0.2249E+00	-0.3490E+00
.1103E+03	0.1171E+01	-0.6552E-01	-0.2327E+00
.1104E+03	0.1555E+01	0.1265E+00	-0.8207E-01
.1105E+03	0.1822E+01	0.3366E+00	0.9797E-01
.1106E+03	0.1859E+01	0.5382E+00	0.2939E+00
.1107E+03	0.1615E+01	0.6971E+00	0.4804E+00
.1108E+03	0.1151E+01	0.7850E+00	0.6308E+00
.1109E+03	0.5101E+00	0.7922E+00	0.7215E+00
.1110E+03	0.4271E+00	0.7299E+00	0.7437E+00
.1111E+03	-0.2343E+00	0.6210E+00	0.7037E+00
.1112E+03	-0.4756E+00	0.4883E+00	0.6174E+00
.1113E+03	-0.6294E+00	0.3488E+00	0.5029E+00
.1114E+03	-0.7231E+00	0.2128E+00	0.3753E+00
.1115E+03	-0.7796E+00	0.8555E-01	0.2457E+00
.1116E+03	-0.8125E+00	-0.3029E-01	0.1209E+00
.1117E+03	-0.8297E+00	-0.1339E+00	0.4776E-02
.1118E+03	-0.8355E+00	-0.2252E+00	-0.1006E+00
.1119E+03	-0.8320E+00	-0.3047E+00	-0.1945E+00
.1120E+03	-0.8194E+00	-0.3725E+00	-0.2768E+00
.1121E+03	-0.7971E+00	-0.4292E+00	-0.3477E+00
.1122E+03	-0.7629E+00	-0.4747E+00	-0.4073E+00
.1123E+03	-0.7135E+00	-0.5087E+00	-0.4556E+00
.1124E+03	-0.6442E+00	-0.5305E+00	-0.4922E+00
.1125E+03	-0.5481E+00	-0.5384E+00	-0.5164E+00
.1126E+03	-0.4154E+00	-0.5302E+00	-0.5268E+00
.1127E+03	-0.2387E+00	-0.5026E+00	-0.5210E+00
.1128E+03	-0.3490E-02	-0.4513E+00	-0.4961E+00

TABLE 4. (CONTD.)

.1129E+03	0.2978E+00	-0.3710E+00	-0.4478E+00
.1130E+03	0.6548E+00	-0.2568E+00	-0.3716E+00
.1131E+03	0.1077E+01	-0.1056E+00	-0.2629E+00
.1132E+03	0.1481E+01	0.3014E-01	-0.1198E+00
.1133E+03	0.1784E+01	0.2886E+00	0.5469E-01
.1134E+03	0.1883E+01	0.4955E+00	0.2488E+00
.1135E+03	0.1699E+01	0.6675E+00	0.4406E+00
.1136E+03	0.1273E+01	0.7736E+00	0.6022E+00
.1137E+03	0.7339E+00	0.7990E+00	0.7081E+00
.1138E+03	0.2278E+00	0.7506E+00	0.7460E+00
.1139E+03	-0.1636E+00	0.6499E+00	0.7189E+00
.1140E+03	-0.4311E+00	0.5207E+00	0.6415E+00
.1141E+03	-0.6015E+00	0.3814E+00	0.5317E+00
.1142E+03	-0.7054E+00	0.2437E+00	0.4058E+00
.1143E+03	-0.7699E+00	0.1140E+00	0.2757E+00
.1144E+03	-0.8072E+00	-0.4657E-02	0.1491E+00
.1145E+03	-0.8275E+00	-0.1112E+00	0.3068E-01
.1146E+03	-0.8357E+00	-0.2054E+00	-0.7737E-01
.1147E+03	-0.8342E+00	-0.2876E+00	-0.1740E+00
.1148E+03	-0.8238E+00	-0.3582E+00	-0.2591E+00
.1149E+03	-0.8039E+00	-0.4175E+00	-0.3326E+00
.1150E+03	-0.7728E+00	-0.4656E+00	-0.3949E+00
.1151E+03	-0.7274E+00	-0.5024E+00	-0.4458E+00
.1152E+03	-0.6634E+00	-0.5272E+00	-0.4853E+00
.1153E+03	-0.5744E+00	-0.5385E+00	-0.5125E+00
.1154E+03	-0.4521E+00	-0.5344E+00	-0.5263E+00
.1155E+03	-0.2853E+00	-0.5118E+00	-0.5247E+00
.1156E+03	-0.5554E-01	-0.4665E+00	-0.5046E+00
.1157E+03	0.2199E+00	-0.3936E+00	-0.4623E+00
.1158E+03	0.5727E+00	-0.2877E+00	-0.3931E+00
.1159E+03	0.9782E+00	-0.1452E+00	-0.2923E+00
.1160E+03	0.1392E+01	0.3323E-01	-0.1572E+00
.1161E+03	0.1730E+01	0.2383E+00	0.1076E-01
.1162E+03	0.1885E+01	0.4488E+00	0.2021E+00
.1163E+03	0.1770E+01	0.6327E+00	0.3973E+00
.1164E+03	0.1393E+01	0.7568E+00	0.5689E+00
.1165E+03	0.8649E+00	0.8015E+00	0.6901E+00
.1166E+03	0.3395E+00	0.7690E+00	0.7445E+00
.1167E+03	-0.8290E-01	0.6783E+00	0.7318E+00
.1168E+03	-0.3783E+00	0.5537E+00	0.6645E+00
.1169E+03	-0.5688E+00	0.4153E+00	0.5606E+00
.1170E+03	-0.6857E+00	0.2753E+00	0.4371E+00
.1171E+03	-0.7583E+00	0.1442E+00	0.3070E+00
.1172E+03	-0.8008E+00	0.2263E-01	0.1789E+00
.1173E+03	-0.8245E+00	-0.8693E-01	0.5815E-01
.1174E+03	-0.8352E+00	-0.1841E+00	-0.5260E-01
.1175E+03	-0.8350E+00	-0.2692E+00	-0.1521E+00
.1176E+03	-0.8277E+00	-0.3426E+00	-0.2400E+00
.1177E+03	-0.8102E+00	-0.4046E+00	-0.3163E+00
.1178E+03	-0.7821E+00	-0.4555E+00	-0.3813E+00
.1179E+03	-0.7405E+00	-0.4951E+00	-0.4350E+00
.1180E+03	-0.6816E+00	-0.5228E+00	-0.4773E+00
.1181E+03	-0.5994E+00	-0.5376E+00	-0.5076E+00
.1182E+03	-0.4851E+00	-0.5376E+00	-0.5249E+00
.1183E+03	-0.3320E+00	-0.5198E+00	-0.5272E+00
.1184E+03	-0.1256E+00	-0.4805E+00	-0.5120E+00
.1185E+03	0.1435E+00	-0.4148E+00	-0.4755E+00
.1186E+03	0.4805E+00	-0.3173E+00	-0.4132E+00
.1187E+03	0.8755E+00	-0.1839E+00	-0.3205E+00
.1188E+03	0.1295E+01	-0.1359E-01	-0.1938E+00
.1189E+03	0.1652E+01	0.1857E+00	-0.3318E-01
.1190E+03	0.1872E+01	0.3989E+00	0.1541E+00
.1191E+03	0.1828E+01	0.5928E+00	0.3510E+00
.1192E+03	0.1507E+01	0.7344E+00	0.5314E+00
.1193E+03	0.1001E+01	0.7993E+00	0.6672E+00
.1194E+03	0.4613E+00	0.7843E+00	0.7386E+00
.1195E+03	-0.8234E-02	0.7054E+00	0.7416E+00
.1196E+03	-0.3175E+00	0.5869E+00	0.6860E+00
.1197E+03	-0.5307E+00	0.4502E+00	0.5892E+00
.1198E+03	-0.6636E+00	0.3103E+00	0.4690E+00
.1199E+03	-0.7445E+00	0.1759E+00	0.3393E+00
.1200E+03	-0.7929E+00	0.5147E-01	0.2101E+00

NOTE: INTERMEDIATE STEPS HAVE BEEN OMITTED.

TABLE 5. LYAPUNOV EXPONENT CALCULATION

X	ELAMDA	ZNORM1	ZNORM2
.7130E+02	0.5028E-01	0.1157E+01	0.9996E+00
.7140E+02	0.4947E-01	0.1139E+01	0.9997E+00
.7150E+02	0.4877E-01	0.1112E+01	0.9997E+00
.7160E+02	0.4801E-01	0.1086E+01	0.9997E+00
.7170E+02	0.4732E-01	0.1052E+01	0.9997E+00
.7180E+02	0.4563E-01	0.1040E+01	0.9997E+00
.7190E+02	0.4593E-01	0.1021E+01	0.9997E+00
.7200E+02	0.4524E-01	0.1004E+01	0.9997E+00
.7210E+02	0.4453E-01	0.9900E+00	0.9997E+00
.7220E+02	0.4388E-01	0.9784E+00	0.9997E+00
.7230E+02	0.4311E-01	0.9694E+00	0.9997E+00
.7240E+02	0.4239E-01	0.9630E+00	0.9997E+00
.7250E+02	0.4167E-01	0.9589E+00	0.9997E+00
.7260E+02	0.4094E-01	0.9570E+00	0.9997E+00
.7270E+02	0.4020E-01	0.9573E+00	0.9997E+00
.7280E+02	0.3947E-01	0.9594E+00	0.9997E+00
.7290E+02	0.3872E-01	0.9633E+00	0.9997E+00
.7300E+02	0.3798E-01	0.9686E+00	0.9997E+00
.7310E+02	0.3723E-01	0.9753E+00	0.9997E+00
.7320E+02	0.3648E-01	0.9832E+00	0.9996E+00
.7330E+02	0.3572E-01	0.9920E+00	0.9996E+00
.7340E+02	0.3496E-01	0.1002E+01	0.9996E+00
.7350E+02	0.3419E-01	0.1012E+01	0.9996E+00
.7360E+02	0.3342E-01	0.1023E+01	0.9996E+00
.7370E+02	0.3265E-01	0.1034E+01	0.9996E+00
.7380E+02	0.3187E-01	0.1045E+01	0.9996E+00
.7390E+02	0.3109E-01	0.1056E+01	0.9996E+00
.7400E+02	0.3031E-01	0.1068E+01	0.9996E+00
.7410E+02	0.2952E-01	0.1079E+01	0.9996E+00
.7420E+02	0.2873E-01	0.1090E+01	0.9996E+00
.7430E+02	0.2795E-01	0.1101E+01	0.9996E+00
.7440E+02	0.2717E-01	0.1111E+01	0.9996E+00
.7450E+02	0.2641E-01	0.1121E+01	0.9996E+00
.7460E+02	0.2568E-01	0.1130E+01	0.9997E+00
.7470E+02	0.2499E-01	0.1139E+01	0.9997E+00
.7480E+02	0.2437E-01	0.1146E+01	0.9997E+00
.7490E+02	0.2384E-01	0.1153E+01	0.9998E+00
.7500E+02	0.2345E-01	0.1159E+01	0.9999E+00
.7510E+02	0.2325E-01	0.1164E+01	0.1000E+01
.7520E+02	0.2330E-01	0.1157E+01	0.1000E+01
.7530E+02	0.2364E-01	0.1159E+01	0.1000E+01
.7540E+02	0.2430E-01	0.1169E+01	0.1000E+01
.7550E+02	0.2532E-01	0.1167E+01	0.1001E+01
.7560E+02	0.2667E-01	0.1153E+01	0.1001E+01
.7570E+02	0.2833E-01	0.1157E+01	0.1001E+01
.7580E+02	0.3025E-01	0.1149E+01	0.1001E+01
.7590E+02	0.3237E-01	0.1138E+01	0.1001E+01
.7600E+02	0.3463E-01	0.1123E+01	0.1001E+01
.7610E+02	0.3700E-01	0.1106E+01	0.1001E+01
.7620E+02	0.3944E-01	0.1084E+01	0.1001E+01
.7630E+02	0.4192E-01	0.1059E+01	0.1001E+01
.7640E+02	0.4442E-01	0.1030E+01	0.1001E+01
.7650E+02	0.4692E-01	0.9957E+00	0.1001E+01
.7660E+02	0.4942E-01	0.9596E+00	0.1001E+01
.7670E+02	0.5190E-01	0.9194E+00	0.1001E+01
.7680E+02	0.5434E-01	0.8775E+00	0.1001E+01
.7690E+02	0.5675E-01	0.8356E+00	0.1001E+01
.7700E+02	0.5912E-01	0.8011E+00	0.1001E+01
.7710E+02	0.6143E-01	0.7780E+00	0.1001E+01
.7720E+02	0.6368E-01	0.7755E+00	0.1001E+01
.7730E+02	0.6584E-01	0.8070E+00	0.1001E+01
.7740E+02	0.6790E-01	0.8783E+00	0.1001E+01
.7750E+02	0.6983E-01	0.9946E+00	0.1001E+01
.7760E+02	0.7163E-01	0.1156E+01	0.1001E+01
.7770E+02	0.7324E-01	0.1358E+01	0.1001E+01
.7780E+02	0.7462E-01	0.1598E+01	0.1001E+01
.7790E+02	0.7573E-01	0.1870E+01	0.1001E+01
.7800E+02	0.7654E-01	0.2156E+01	0.1000E+01
.7810E+02	0.7678E-01	0.2483E+01	0.1000E+01
.7820E+02	0.7650E-01	0.2805E+01	0.9997E+00
.7830E+02	0.7544E-01	0.3121E+01	0.9992E+00
.7840E+02	0.7333E-01	0.3412E+01	0.9986E+00
.7850E+02	0.6983E-01	0.3650E+01	0.9978E+00
.7860E+02	0.6495E-01	0.3842E+01	0.9973E+00
.7870E+02	0.6181E-01	0.3937E+01	0.1000E+01
.7880E+02	0.6501E-01	0.3928E+01	0.1004E+01
.7890E+02	0.7276E-01	0.3808E+01	0.1003E+01
.7900E+02	0.7795E-01	0.3581E+01	0.1002E+01
.7910E+02	0.8124E-01	0.3268E+01	0.1001E+01
.7920E+02	0.8286E-01	0.2903E+01	0.1001E+01
.7930E+02	0.8306E-01	0.2527E+01	0.9998E+00
.7940E+02	0.8205E-01	0.2183E+01	0.9992E+00
.7950E+02	0.8004E-01	0.1932E+01	0.9987E+00
.7960E+02	0.7720E-01	0.1700E+01	0.9983E+00
.7970E+02	0.7377E-01	0.1571E+01	0.9981E+00
.7980E+02	0.6994E-01	0.1497E+01	0.9979E+00
.7990E+02	0.6597E-01	0.1456E+01	0.9979E+00
.8000E+02	0.6210E-01	0.1431E+01	0.9980E+00

TABLES: (CONTD.)

X	ELAMDA	ZNORM1	ZNORM2
.1000E+03	0.0000E+00	0.1149E+01	0.7553E+00
.2000E+03	0.7044E-02	0.1555E+01	0.1640E+00
.3000E+03	0.1954E-02	0.1295E+01	0.4443E+01
.4000E+03	0.4944E-02	0.2205E+01	0.9706E-01
.5000E+03	-0.5309E-04	0.1017E+01	0.7099E+00
.6000E+03	-0.2990E-02	0.1381E+01	0.7004E+00
.7000E+03	0.2453E-02	0.1125E+01	0.5420E+01
.8000E+03	0.3535E-02	0.2475E+01	0.6691E+00
.9000E+03	0.2575E-03	0.9999E+00	0.7105E+00
.1000E+04	-0.1374E-02	0.1326E+01	0.7053E+00
.1100E+04	0.1915E-03	0.1343E+01	0.4153E+01
.1200E+04	0.4321E-02	0.3855E+01	0.1559E+01
.1300E+04	0.3840E-03	0.1349E+01	0.7122E+00
.1400E+04	-0.7560E-03	0.1262E+01	0.7077E+00
.1500E+04	-0.7549E-03	0.1439E+01	0.1865E+01
.1600E+04	0.1388E-02	0.7508E+01	0.6596E+00
.1700E+04	0.4415E-03	0.1194E+01	0.7174E+00
.1800E+04	-0.4342E-03	0.1191E+01	0.7088E+00
.1900E+04	-0.4082E-02	0.1451E+01	0.8719E+00
.2000E+04	0.2159E-02	0.4822E+01	0.3450E+01
.2100E+04	0.4745E-03	0.1149E+01	0.6979E+00
.2200E+04	-0.2309E-03	0.1118E+01	0.7093E+00
.2300E+04	-0.8913E-03	0.1446E+01	0.7009E+00
.2400E+04	0.1457E-02	0.1891E+01	0.4953E+01
.2500E+04	0.4931E-03	0.1814E+01	0.2347E+00
.2600E+04	-0.9503E-04	0.1050E+01	0.7097E+00
.2700E+04	-0.6591E-03	0.1411E+01	0.6959E+00
.2800E+04	0.8158E-03	0.1008E+01	0.5441E+01
.2900E+04	0.6313E-03	0.2248E+01	0.1014E+00
.3000E+04	0.3203E-05	0.1006E+01	0.7101E+00
.3100E+04	-0.4328E-03	0.1352E+01	0.7026E+00
.3200E+04	0.3046E-03	0.1221E+01	0.5192E+01
.3300E+04	0.1271E-02	0.2604E+01	0.9588E+00
.3400E+04	0.7748E-04	0.1099E+01	0.7110E+00
.3500E+04	-0.3505E-03	0.1303E+01	0.7064E+00
.3600E+04	-0.9578E-04	0.1391E+01	0.3238E+01
.3700E+04	0.1312E-02	0.6592E+01	0.1754E+01
.3800E+04	0.1351E-03	0.1091E+01	0.7135E+00
.3900E+04	-0.2475E-03	0.1236E+01	0.7082E+00
.4000E+04	-0.3877E-03	0.1454E+01	0.1334E+01
.4100E+04	0.9350E-04	0.6978E+01	0.1557E+01
.4200E+04	-0.4319E-03	0.1276E+01	0.7203E+00
.4300E+04	-0.4547E-03	0.1163E+01	0.7090E+00
.4400E+04	-0.4787E-03	0.1459E+01	0.7615E+00
.4500E+04	0.3758E-03	0.3519E+01	0.4196E+01
.4600E+04	0.2192E-03	0.1573E+01	0.5846E+00
.4700E+04	-0.9556E-04	0.1091E+01	0.7095E+00
.4800E+04	-0.4105E-03	0.1435E+01	0.6942E+00
.4900E+04	0.6029E-03	0.3209E+01	0.5210E+01
.5000E+04	0.2484E-03	0.1973E+01	0.1302E+00
.5100E+04	-0.3944E-04	0.1030E+01	0.7098E+00
.5200E+04	-0.3255E-03	0.1394E+01	0.6986E+00
.5300E+04	0.3248E-03	0.3051E+01	0.5468E+01
.5400E+04	0.3477E-03	0.2394E+01	0.2186E+00
.5500E+04	0.9190E-05	0.9995E+00	0.7103E+00
.5600E+04	-0.2551E-03	0.1341E+01	0.7044E+00
.5700E+04	0.7399E-04	0.1300E+01	0.4684E+01
.5800E+04	0.7491E-04	0.3048E+01	0.1237E+01
.5900E+04	0.5397E-04	0.1026E+01	0.7116E+00
.6000E+04	-0.4948E-03	0.1279E+01	0.7073E+00

A=9

TABLE 5. (CONTD.)

X	ELAMDA	ZNORM1	ZNORM2
.1000E+03	0.0000E+00	0.8604E+00	0.4476E+00
.2000E+03	-0.4284E-01	0.9813E+00	0.4173E+00
.3000E+03	0.2251E-02	0.1980E+01	0.3241E+01
.4000E+03	-0.4009E-02	0.8253E+00	0.5864E+00
.5000E+03	-0.1088E-02	0.8530E+00	0.9291E+00
.6000E+03	0.2305E-02	0.1573E+01	0.3409E+01
.7000E+03	-0.4125E-02	0.8778E+00	0.2337E+00
.8000E+03	0.6983E-03	0.6536E+00	0.2292E+01
.9000E+03	-0.1281E-02	0.1047E+01	0.1394E+01
.1000E+04	-0.1240E-02	0.8551E+00	0.2376E+00
.1100E+04	0.1109E-02	0.9158E+00	0.4790E+01
.1200E+04	0.1958E-03	0.9151E+00	0.5304E+00
.1300E+04	-0.1216E-02	0.9957E+00	0.3223E+00
.1400E+04	0.7315E-03	0.1708E+01	0.5339E+01
.1500E+04	-0.1060E-03	0.8350E+00	0.4885E+00
.1600E+04	-0.7020E-03	0.9517E+00	0.5430E+00
.1700E+04	-0.4832E-03	0.1936E+01	0.3402E+01
.1800E+04	-0.2772E-03	0.8375E+00	0.3940E+00
.1900E+04	-0.5797E-04	0.7850E+00	0.1316E+01
.2000E+04	0.7246E-03	0.1311E+01	0.2665E+01
.2100E+04	-0.4368E-03	0.8080E+00	0.2140E+00
.2200E+04	0.3522E-03	0.6592E+00	0.3133E+01
.2300E+04	0.2914E-03	0.9812E+00	0.9041E+00
.2400E+04	-0.5734E-03	0.9776E+00	0.2693E+00
.2500E+04	0.4770E-03	0.1205E+01	0.5638E+01
.2600E+04	0.3225E-05	0.8799E+00	0.4560E+00
.2700E+04	-0.5520E-03	0.9915E+00	0.3683E+00
.2800E+04	0.1950E-03	0.1920E+01	0.3906E+01
.2900E+04	-0.1070E-03	0.8244E+00	0.5928E+00
.3000E+04	-0.2569E-03	0.9036E+00	0.7473E+00
.3100E+04	0.3945E-03	0.1742E+01	0.3632E+01
.3200E+04	-0.2104E-03	0.8501E+00	0.2699E+00
.3300E+04	0.6498E-04	0.7042E+00	0.1855E+01
.3400E+04	0.3468E-03	0.1121E+01	0.1834E+01
.3500E+04	-0.3178E-03	0.9378E+00	0.2224E+00
.3600E+04	0.2560E-03	0.7759E+00	0.4132E+01
.3700E+04	0.8130E-04	0.9399E+00	0.6280E+00
.3800E+04	-0.4023E-03	0.9919E+00	0.3016E+00
.3900E+04	0.2710E-03	0.1522E+01	0.5810E+01
.4000E+04	-0.3901E-04	0.6491E+00	0.4529E+00

REFERENCES

1. M. Ash : "Nuclear Reactor Kinetics"; Mc Graw-Hill (1979).

2. J. Lewins : "Nuclear Reactor Kinetics and Control" ; Pergamon Press (1978).

3. D.L. Hetrick : "Dynamics of Nuclear Reactors" ; Univ. of Chicago Press (1971).

4. A.Z. Akcasu, C.S. Lellouche, L.M. Shotkin : "Mathematical Methods in Nuclear Reactor Dynamics" ; Academic Press (1971).

5. A.Z. Akcasu, L.M. Shotkin : "Power Oscillations and the Describing Function in Reactors with Linear Feedback" ; Nucl. Sci. & Engg., 28, 72-81 (1967).

6. J. Devooght, H.B. Smets : "Determination of Stability Domains in Point Reactor Dynamics"; Nucl. Sci. & Engg., 28, 226-236 (1967).

7. T.R. Schmidt, D.L. Hetrick : "Nonlinear Oscillations and Stability of a Nuclear Reactor with Two Reactivity Feedbacks ; Nucl. Sci. & Engg., 42, 1-9 (1970).

8. L.M. Shotkin, D.L. Hetrick, T.R. Schmidt : "Effect of Delayed Neutrons on Autonomous Nonlinear Power Oscillations" ; Nucl. Sci. & Engg., 42, 10-15 (1970).

9. S.A. Vreeke, G.M. Sandquist : "Phase Space Analysis of Reactor Kinetics" ; Nucl. Sci. & Engg., 42, 295-305 (1970).

10. J.P. Keener, D.S. Cohen : "Nonlinear Oscillations in a Reactor with Two Temperature Coefficients" ; Nucl. Sci. & Engg., 56, 354-359 (1975).

11. R.S. Poddar, M.S. Trasi, V. Balaraman : "Power Oscillations in a Reactor with Two Temperature Coefficients" ; Atomkernenergie Kerntechnik 42, 95-100 (1983).

12. M. Ashok Kumar : "Two Temperature Reactor Model Delayed Neutrons and Feedback Effects" ; M. Tech. Thesis, Nuclear Engg. & Tech. Prog., Indian Inst. Tech., Kanpur, India (1984).

13. T.B. Enginol : "On the Asymptotic Stability of Nuclear Reactors with Arbitrary Feedback" ; Nucl. Sci. & Engg., 90, 231-235 (1985).

14. D.W.Jordan, P. Smith : "Nonlinear Ordinary Differential Equations" ; Oxford Univ. Press (1987).

15. E. Beltrami : "Mathematics for Dynamic Modelling"; Academic Press (1987).

16. G. Iooss, D.D. Joseph : "Elementary Stability and Bifurcation Theory" ; Springer-Verlag (1980).

17. M. Kubicek, M. Marek : "Computational Methods in Bifurcation Theory and Dissipative Structures"; Springer-Verlag (1983).

18. J.E. Marsden, M. McCracken : "The Hopf Bifurcation and Its Applications" ; Springer-Verlag (1976).

19. B.D. Hassard, N.D. Kazarinoff, Y-H. Wan : "Theory and Applications of Hopf Bifurcation" ; Cambridge Univ. Press (1981).

20. M. Holodinoik, M. Kubicek : "New Algorithms for the Evaluation of Complex Bifurcation Points in Ordinary Differential Equations - A Comparative Numerical Study" ; Appl. Maths. & Comput., 15, 261-274 (1984).

21. J.M.T. Thompson, H.B. Stewart : "Nonlinear Dynamics and Chaos" ; John Wiley (1986).

22. P. Berge, Y. Poomeau, C. Vidal : "Order within Chaos" ; John Wiley (1984).

23. F.C. Moon : "Chaotic Vibrations" ; John Wiley (1987).

24. R. Devany : "Introduction to Chaotic Dynamical Systems" ; Benjamin Cummings (1986).

25. G. Hall, J.M. Watt (eds.) : "Modern Numerical Methods for Ordinary Differential Equations" ; Clerendon Press (1976).

26. E.V. Krishnamurthy, S.K. Sen : "Numerical Algorithms" ; Affiliated East-West Press (1986).

27. M.K. Jain, S.R.K. Iyengar, R.K. Jain : "Numerical Methods for Scientific and Engineering Computation" ; Wiley Eastern (1985).

28. J. Wolf, J.B. Swift, H.L. Swinney, J.A. Vastano : "Determining Lyapunov Exponents from a Time Series" ; Physica 16D, 285-317 (1985).

29. I. Shimada, T. Nagashima : "A Numerical Approach to Ergodic Problems of Dissipative Dynamical Systems"; prog. Theor. Physics, 61, 1605-1615 (1979).

30. J.M. Greene, J.-S. Kim : "The Calculation of Lyapunov Spectra" ; Physica 24D, 213-225 (1987).